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# Fluxes, moduli fixing and MSSM-like vacua in a simple IIA orientifold

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Abstract: We study the effects of adding RR, NS and metric fluxes on a $\mathrm{T}^6/(\Omega(-1)^{F_L} I_3)$ type IIA orientifold. By using the effective flux-induced superpotential we obtain Minkowski or AdS vacua with broken or unbroken supersymmetry. In the Minkowski case some combinations of real moduli remain undetermined, whereas all can be stabilized in the AdS solutions. Many flux parameters are available which are unconstrained by RR tadpole cancellation conditions allowing to locate the minima at large volume and small dilaton. We also find that in AdS supersymmetric vacua with metric fluxes, the overall flux contribution to RR tadpoles can vanish or have opposite sign to that of D6-branes, allowing for new model-building possibilities. In particular, we construct the first $\mathcal{N}$=1 supersymmetric intersecting D6-brane models with MSSM-like spectrum and with all closed string moduli stabilized. Some axion-like fields remain undetermined but they are precisely required to give Stückelberg masses to (potentially anomalous) U(1) brane fields. We show that the cancellation of the Freed-Witten anomaly guarantees that the axions with flux-induced masses are orthogonal to those giving masses to the U(1)'s. Cancellation of such anomalies also guarantees that the D6-branes in our $\mathcal{N}$=1 supersymmetric AdS vacua are calibrated so that they are forced to preserve one unbroken supersymmetry.

Keywords: D-branes, Superstring Vacua.

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## Contents





## 1. Introduction

One of the most pressing problems in string theory is the issue of moduli stabilization. Lately, important progress has been accomplished by taking into account the freedom of switching on (quantized) RR and NS fluxes in the compact closed string background. This road has been particularly explored in the context of type IIB theory, where RR/NS fluxes create a superpotential [1] that depends on the complex structure fields and the axi-dilaton and allows to fix these fields dynamically [2 – 13]. In order to further determine the Kähler moduli, non-perturbative effects have also been put to work [14, 15]. For other proposals for fixing Kähler moduli, see [16]. In simple IIB toroidal orientifolds [3 – 7, 10 – 13] in general the moduli are fixed in regions in which the compact volume is of order the string scale and/or the dilaton is of order one, so that the validity of an effective 4-dimensional supergravity action is open to question. One of the important reasons why this is the case is that the values of fluxes are strongly constrained by RR tadpole cancellation conditions. The situation is ameliorated in compactifications on IIB Calabi-Yau orientifolds since the flux contribution to tadpoles is typically large and one can generate vacua with all moduli and the dilaton in regions of parameter space where the effective supergravity approximation may be trusted [9].

In comparison, less effort has been devoted to the similar moduli-fixing problem in the case of type IIA compactifications. The fact that in type IIA there are fluxes with both even and odd rank suggests that both complex structure and Kähler moduli fields may be determined simultaneously without resorting to non-perturbative effects. This has been anticipated by several authors [17 – 21]. Indeed, in the type IIA case, RR/NS backgrounds give rise to superpotentials depending both on Kähler and complex structure moduli, but with no terms mixing both kinds of moduli. Furthermore, in simple toroidal settings one can also include metric fluxes and generate superpotential terms coupling both kinds of moduli [17, 20]. The so-called metric fluxes can arise partially from T-duality of NS fluxes [22 – 24]. More generally, turning on constant metric fluxes corresponds to Scherk-Schwarz reductions [25] that can be understood as compactifications on twisted tori [26 – 29].

The purpose of this paper is twofold. We will first present a detailed study of minima of the moduli potential induced by RR, NS and metric fluxes in the simple $\mathrm{T}^6/(\Omega(-1)^{F_L}I_3)$ type IIA orientifold. We concentrate on the potential for the dilaton and the diagonal Kähler and complex structure moduli, which may be also viewed as the only untwisted moduli of a related $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold. We argue though that the results found for $\mathrm{T}^6/(\Omega(-1)^{F_L}I_3)$ ignoring off-diagonal moduli still constitute extrema of the potentials which are stable in relevant cases. We find four classes of (non-singular) vacua which correspond to $\mathcal{N}$=1 supersymmetric models in Minkowski space, no-scale, AdS with $\mathcal{N}$=1 supersymmetry and non-supersymmetric AdS models. In the Minkowski cases only a few of the moduli may be determined. On the other hand, the AdS vacua look particularly interesting since all moduli are stabilized (except for a combination of axion-like fields, we come back to this point below).

The structure of vacua in both Minkowski and AdS space depends very much on the existence or not of metric fluxes which lead to some remarkable new features. In particular, in $\mathcal{N}$=1 supersymmetric AdS vacua *without* metric fluxes, NS and RR fluxes always contribute to RR tadpoles like D6-branes do. The RR tadpole cancellation conditions restrict some of the flux parameters but some others (particularly the RR 4-form and 2-form fluxes) remain unconstrained. Due to this fact, one can easily find minima with all closed string moduli stabilized in regions with large volume and small dilaton, so that the effective supergravity action should be a good approximation. The (negative) cosmological constant may be made arbitrarily small for sufficiently large fluxes. $\mathcal{N}$=1 supersymmetric AdS min-



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ima in generic IIA orientifolds with NS and RR fluxes were recently analyzed in [21]. In this case we obtain analogous results. We also find examples of AdS vacua with broken supersymmetry and all moduli stabilized.

In the $\mathcal{N}$=1 AdS vacua with RR/NS backgrounds *and* metric fluxes turned on a particular new property appears. The flux contribution to the RR tadpoles may be positive, negative or zero. This is due to the fact that the RR-charge $Q_{RR}$ has the schematic structure $Q_{RR} \simeq (m\overline{H}_3 + \omega\overline{F}_2)$, where $m$ is the 0-form of massive IIA supergravity and $\omega$ represents the metric flux parameters. The signs of the different fluxes are not arbitrary since they are correlated to the signs of the real parts of the moduli fixed at the minimum. The fact that we can add fluxes determining moduli but not contributing to RR tadpoles is important since this means that we have a rigid 'corset', namely the concrete AdS $\mathcal{N}$=1 background, which can be added to any RR tadpole-free configuration of D6-branes to stabilize all moduli. On the other hand, considering fluxes contributing like O6-planes to RR tadpoles is interesting since we can dispose of orientifold planes in certain cases. We find, at least for the massive $m \neq 0$ case, that the AdS supersymmetric minima may be made to reside at points with large compact volume and small dilaton so that corrections are under control.

The observation that one can have string backgrounds leading to vanishing or negative RR charges in AdS is not new, see e.g. [30, 31], and is also related to the fact that in the presence of metric fluxes one is really dealing with non-Calabi-Yau manifolds with peculiar topology. In our case we have a twisted torus with a half-flat structure. Then, a D6-brane wrapping a certain 3-cycle in the original torus with RR charges cancelled by some background including metric fluxes may be alternatively understood as a homologically trivial brane in the twisted torus which is however stable because it wraps a generalized calibrated 3-cycle [31, 32].

The second main topic in this paper is the inclusion of D6-branes in models with fluxes and the construction of some semi-realistic examples. It turns out that adding branes gives rise to some new interesting features beyond the obvious one of their contribution to RR tadpoles. Stacks of D6-branes wrapping 3-cycles contain in general U(1) fields which couple to RR fields, the imaginary parts of the complex structure moduli and the axi-dilaton. In particular, some of the U(1)'s get Stückelberg masses by combining with these RR fields. Now, if some of such RR fields get masses from fluxes some inconsistency is expected, in particular, the flux induced superpotential would violate gauge invariance. Therefore, we need that the linear combinations of RR fields combining with U(1)'s and those getting masses from fluxes should be orthogonal. We find that this is guaranteed as long as the Freed-Witten anomaly [33, 34] induced on the world-volume of D6-branes by the fluxes cancels. This turns out to be an important constraint on the Minkowski minima. In the case of AdS $\mathcal{N}$=1 supersymmetric minima, with the real parts of all moduli determined, we find the interesting result that the cancellation of the FW anomaly automatically forces the branes to preserve supersymmetry, i.e. to wrap special Lagrangian (slag) cycles. Stating it the other way around, any D6-brane wrapping slag cycles will automatically be free of the FW anomaly in this background.

It is important to see how far one can go in stabilizing all moduli in models with possible phenomenological relevance. We present examples of configurations of D6-branes wrapping 3-cycles on the torus and intersecting at angles with chiral MSSM-like spectra and fixed moduli. The models contain three generations of quarks and leptons and one Higgs set, with the gauge group of the SM extended by one or two extra U(1)'s and some extra heavy vector-like $\mathrm{SU}(2)_L$ doublets and singlets. Some of the examples live in Minkowski space (either $\mathcal{N}=1$ supersymmetric or no-scale), in which case only a few moduli are fixed. On the other hand, we present the first semi-realistic $\mathcal{N}=1$ supersymmetric model in AdS with all closed string moduli stabilized. This model requires the presence of both metric and NS/RR fluxes so that a 'wrong sign' contribution to tadpoles, mimicking orientifold planes, is obtained. The minima are stabilized in a perturbative regime and all physical quantities like gauge and Yukawa couplings are given in terms of otherwise undetermined NS and RR fluxes.

The outline of the paper is as follows. In the next section 2 we introduce the basic tools needed to describe the $\mathrm{T}^6/(\Omega(-1)^{F_L}I_3)$ type IIA orientifold with fluxes. In section 3 we display the structure of the flux-induced superpotential and discuss the contribution of different fluxes to RR charges. The systematic analysis of the different vacua of the flux-induced potential is presented in section 4. Examples of Minkowski and AdS vacua with/without supersymmetry and with/without metric fluxes are reported. In section 5 we examine the constraints coming from the Freed-Witten anomaly and their connection to the open string U(1)'s. Specific semi-realistic intersecting D6-brane models with MSSM-like spectrum are discussed in section 6. This includes the AdS $\mathcal{N}=1$ supersymmetric example with all closed string moduli stabilized. We present some comments and conclusions in section 7. Some related results are collected in two appendices. In appendix A we study the SU(3) structure of the twisted torus and discuss $\mathcal{N}=1$ vacua in terms of torsion classes. In appendix B we present some non-supersymmetric D6-brane configurations with moduli stabilized in AdS.

## 2. Basic features

The aim of this section is to present the concepts needed to describe the low-energy effective action of type IIA orientifolds in the presence of background fluxes. We first introduce the moduli fields and then exhibit the superpotential induced by NS and RR fluxes. We next define the so-called metric fluxes and recall how they can partially arise from T-duality of NS fluxes in type IIB.

### 2.1 IIA orientifolds: moduli and NS/RR fluxes

This section contains a brief review of the structure of IIA Calabi-Yau orientifolds. The treatment follows [19] which the reader can consult for more details. We limit to a discussion of moduli fields, NS/RR fluxes, and flux-induced superpotentials. Our main purpose is to apply the general results to a simple IIA toroidal orientifold.

Compactification of type IIA strings on a Calabi-Yau 3-fold $Y$ gives a $D=4$, $\mathcal{N}=2$ theory with $h_{11}$ vector multiplets and $(1 + h_{12})$ hypermultiplets [35]. Turning on fluxes for the NS and RR field strengths generates a potential for the scalars in these multiplets [36, 37, 18]. To obtain a $\mathcal{N}$=1 theory one can implement an orientifold projection by $\Omega_P(-1)^{F_L}\sigma$, where $\Omega_P$ is the world-sheet parity operator, $(-1)^{F_L}$ is the space-time fermionic number for left-movers and $\sigma$ is an order two involution of $Y$. The action of $\sigma$ on the Kähler form and the holomorphic 3-form is $\sigma(J) = -J$ and $\sigma(\Omega) = e^{2i\theta}\Omega^*$. We take $\theta = 0$ and $\sigma(z^i) = \bar{z}^i$, where $z^i$ are local complex coordinates. This implies O6-planes whose tadpoles can be cancelled by adding D6-branes or flux, as we will see.

The closed (1,1) forms split into $h_{11}^+$ and $h_{11}^-$, according to whether they are even or odd under $\sigma$. There is an equal number $(1 + h_{12})$ of even and odd 3-forms. Then, the resulting matter content from the closed string sector consists of $h_{11}^+$ vector multiplets, $h_{11}^-$ chiral multiplets corresponding to Kähler moduli, and $(1 + h_{12})$ chiral multiplets corresponding to the dilaton and the complex structure moduli [19]. The scalar components of the Kähler moduli, denoted $T_A$, are defined in terms of the complexified Kähler form as

$$J_c = B + iJ = i\sum_{A=1}^{h_{11}^-} T_A\omega_A\,, \tag{2.1}$$

where $\omega_A$ are the $\sigma$-odd (1,1) closed forms. The complex structure moduli, denoted $N_L$, $L = 0, \cdots, h_{12}$, can be extracted from

$$iN_L = \int_Y \Omega_c \wedge \beta_L\,; \qquad \Omega_c = C_3 + i\mathrm{Re}(C\Omega)\,, \tag{2.2}$$

where $C_3 = \sum_L \xi_L\alpha_L$ provides the axions. Here $\alpha_L$ and $\beta_L$ are respectively the $\sigma$-even and $\sigma$-odd 3-forms. The field $C$ is in turn specified by

$$C = e^{-\phi_4}e^{K_{cs}/2}\,; \qquad K_{cs} = -\log\left[-\frac{i}{8}\int_Y \Omega\wedge\Omega^*\right], \tag{2.3}$$

where $\phi_4$ is the T-duality invariant four-dimensional dilaton given by $e^{\phi_4} = e^{\phi_{10}}/\sqrt{\mathrm{vol}\,Y}$.

To be more explicit let us now consider the example of a factorized 6-torus $\otimes_{j=1}^3 \mathrm{T}_j^2$. The area of each sub-torus is $(2\pi)^2 A_j$, where $A_j = R_x^j R_y^j$. The $A_j/\alpha'$ are thus the real part, $t_j$, of three Kähler moduli $T_j$. For each sub-torus we take a square lattice, consistent with the orientifold projection. The complex structure parameter of each $\mathrm{T}_j^2$ is then $\tau_j = R_y^j/R_x^j$. It is known, see e.g. [38], that in this setup the IIA $D$=4 fields $S$ and $U_i$, corresponding to the dilaton and complex structure moduli, have real parts

$$\mathrm{Re}S \equiv s = \frac{e^{-\phi_4}}{\sqrt{\tau_1\tau_2\tau_3}}\,, \qquad \mathrm{Re}U_i \equiv u_i = e^{-\phi_4}\sqrt{\frac{\tau_j\tau_k}{\tau_i}}\,, \qquad i \neq j \neq k\,, \tag{2.4}$$

where $e^{\phi_4} = e^{\phi}/\sqrt{t_1t_2t_3}$. We will next obtain these results from the general analysis of [19]. As usual, the holomorphic 3-form can be written as

$$\Omega = \left(dx^1 + i\tau_1\,dy^1\right)\wedge\left(dx^2 + i\tau_2\,dy^2\right)\wedge\left(dx^3 + i\tau_3\,dy^3\right), \tag{2.5}$$

where $y^i = x^{i+3}$. The orientifold involution acts as $\sigma(x^i) = x^i$ and $\sigma(y^i) = -y^i$. The even and odd 3-forms with one leg on each sub-torus are

$$\begin{aligned}
\alpha_0 &= dx^1 \wedge dx^2 \wedge dx^3\,, \qquad & \beta_0 &= dy^1 \wedge dy^2 \wedge dy^3\,, \\
\alpha_1 &= dx^1 \wedge dy^2 \wedge dy^3\,, \qquad & \beta_1 &= dy^1 \wedge dx^2 \wedge dx^3\,, \\
\alpha_2 &= dy^1 \wedge dx^2 \wedge dy^3\,, \qquad & \beta_2 &= dx^1 \wedge dy^2 \wedge dx^3\,, \\
\alpha_3 &= dy^1 \wedge dy^2 \wedge dx^3\,, \qquad & \beta_3 &= dx^1 \wedge dx^2 \wedge dy^3\,.
\end{aligned} \tag{2.6}$$

Our normalization is such that $\int_{\mathrm{T}^6} \alpha_I \wedge \beta_J = \delta_{IJ}$. Substituting in (2.3) we find $C = \mathrm{Re}S$. From (2.2) we then obtain the corresponding moduli $N_0 = S$ and $N_i = -U_i$.

We have only considered 3-forms with one leg on each $\mathrm{T}^2_j$ because these are the directions in which we are going to switch on fluxes. If the orientifold has an extra $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry these are in fact the only invariant forms.

The next step is to turn on background fluxes. The NS $H_3$ is odd under the orientifold action, thus the general flux allowed is

$$\overline{H}_3 = \sum_{L=0}^{h_{12}} h_L \beta_L\,. \tag{2.7}$$

For the RR forms, $F_0$ and $F_4$ are even while $F_2$ and $F_6$ are odd under the orientifold projection. We can then have the general expansions

$$\begin{aligned}
\overline{F}_0 &= -m\,; \qquad & \overline{F}_6 &= e_0 d\mathrm{vol}_6 \\
\overline{F}_2 &= \sum_{A=1}^{h_{11}^-} q_A \omega_A\,; \qquad & \overline{F}_4 &= \sum_{A=1}^{h_{11}^-} e_A \widetilde{\omega}_A\,,
\end{aligned} \tag{2.8}$$

where $\widetilde{\omega}_A$ are the $h_{22}^+ = h_{11}^-$ $\sigma$-even (2,2) forms. There are also quantization conditions

$$\frac{\ell^3 \mu_1}{2\pi} \int_{\Pi_3} \overline{H}_3 \in \mathbb{Z}\,, \qquad \frac{\ell^p \mu_{p-2}}{2\pi} \int_{\Pi_p} \overline{F}_p \in \mathbb{Z}\,, \tag{2.9}$$

for any $p$-cycle $\Pi_p$ in $Y$. Here $\ell = 2\pi\sqrt{\alpha'}$, and $\mu_p = 1/(2\pi)^p \alpha'^{\,(p+1)/2}$ [39, 40]. We normally take the various forms, e.g. $\beta_L$, to belong to an integer basis so that in units of $2\pi/\mu_{p-2}\ell^p = 1/\ell$ the various coefficients, such as $h_L$, are integers. Actually, to avoid subtleties with exotic orientifold planes [2, 3] we take the coefficients to be even. Notice that by including the factors of $\ell$ explicitly, all forms have dimensions $(\mathrm{length})^{-1}$. With these conventions the moduli fields are all dimensionless.

The RR fluxes generate a superpotential for the Kähler moduli that can be written as

$$W_K = \int_Y e^{J_c} \wedge \overline{F}_{RR}\,, \tag{2.10}$$

where $\overline{F}_{RR}$ represents a formal sum of the even RR fluxes. This result can be obtained [41] applying mirror symmetry to the type IIB superpotential [1]. It can also be derived performing the explicit Kaluza-Klein reduction [19] which also allows to determine the superpotential for the complex structure moduli due to NS flux, namely

$$W_Q = \int_Y \Omega_c \wedge \overline{H}_3 = i \sum_{L=0}^{h_{12}} h_L N_L\,, \tag{2.11}$$

where we have used (2.2). The Kähler potential for both kinds of moduli are given by

$$K_K = -\log\left[\frac{4}{3}\int_Y J\wedge J\wedge J\right]\,, \qquad K_Q = -\log e^{-4\phi_4}\,. \tag{2.12}$$

There are corrections to $K_K$ due to world-sheet instantons and to $K_Q$ due to D2 instantons [19]. We will see later that one can locate the minima of the potential in regions with large volume and small dilaton in which these corrections should be in principle under control.

It is instructive to apply these results to the $\otimes_{j=1}^3 \mathrm{T}_j^2$ example. The $\beta_L$ are given in (2.6) whereas

$$\omega_i = -dx^i\wedge dy^i\,, \qquad \widetilde{\omega}_i = dx^j\wedge dy^j\wedge dx^k\wedge dy^k\,, \qquad i\neq j\neq k\,. \tag{2.13}$$

Notice that $\int_{\mathrm{T}^6}\omega_i\wedge\widetilde{\omega}_j = \delta_{ij}$. It is straightforward to substitute the flux expansions in (2.10) and (2.11) to obtain

$$\begin{aligned} W_K &= e_0 + i\sum_{i=1}^{3} e_i T_i - q_1 T_2 T_3 - q_2 T_1 T_3 - q_3 T_1 T_2 + imT_1T_2T_3\,,\\ W_Q &= ih_0 S - i\sum_{i=1}^{3} h_i U_i\,. \end{aligned} \tag{2.14}$$

These superpotentials have been recently discussed in [17, 20]. Finally, the Kähler potential takes the usual expression

$$K = -\log(S+S^*) - \sum_{i=1}^{3}\log(U_i+U_i^*) - \sum_{i=1}^{3}\log(T_i+T_i^*)\,. \tag{2.15}$$

### 2.2 Metric fluxes and twisted tori

In the next section we will see how superpotential terms mixing Kähler and complex structure moduli, including the dilaton, can be generated by switching on so-called metric fluxes. Such backgrounds appear naturally in the context of Scherk-Schwarz reductions [25]. In turn these can be shown (see e.g. [26]) to be equivalent to compactification on a twisted torus defined by

$$d\eta^P = -\frac{1}{2}\omega^P_{MN}\eta^M\wedge\eta^N\,, \tag{2.16}$$

where $\omega^P_{MN}$ are constant coefficients, antisymmetric in the lower indices. The structure constants $\omega^P_{MN}$ are the metric fluxes we are interested in. The $\eta^P$ are the tangent 1-forms and can depend linearly on the internal coordinates $x^M$, concretely

$$\eta^M = N_N{}^M(x)dx^N\,, \qquad dx^N = N^N{}_M(x)\eta^M\,. \tag{2.17}$$

One can define isometry generators as

$$Z_M = N^N{}_M\frac{\partial}{\partial x^N}\,. \tag{2.18}$$

The metric fluxes are actually the structure constants of the Lie algebra generated by the $Z_M$, i.e.

$$[Z_M, Z_N] = \omega^P_{MN} Z_P\,. \tag{2.19}$$

Either from the Jacobi identity of the algebra or from the Bianchi identity of (2.16) one finds that the metric fluxes must satisfy

$$\omega^P_{[MN}\omega^S_{R]P} = 0\,. \tag{2.20}$$

It can further be shown that $\omega^P_{PN} = 0$ [25].

We can derive a helpful result for the exterior derivative of a 2-form $X = X_{MN}\eta^M \wedge \eta^N$ using (2.16). For coefficients independent of the $x^N$ we readily find

$$(dX)_{LMN} = \omega^P_{[LM}X_{N]P}\,. \tag{2.21}$$

Similar formulas can be obtained for higher forms.

We will focus on the case in which only metric fluxes of type $\omega^i_{ab}$, $\omega^i_{jk}$, $\omega^a_{ib}$, $i = 1,2,3$, $a = 4,5,6$, are allowed. This can be implemented by imposing a symmetry of (2.16) under the orientifold involution $\eta^i \to \eta^i$, $\eta^a \to -\eta^a$. As in the case of RR and NS fluxes discussed previously, we are only going to switch on metric fluxes along factorized directions. These correspond to the structure constants which are invariant under a $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry whose generators transform $(\eta^1, \eta^2, \eta^3, \eta^4, \eta^5, \eta^6)$ into $(-\eta^1, -\eta^2, \eta^3, -\eta^4, -\eta^5, \eta^6)$ and $(\eta^1, -\eta^2, -\eta^3, \eta^4, -\eta^5, -\eta^6)$. In the end there are twelve metric fluxes left. To write down the relations that follow from (2.20) we introduce the notation

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} \omega^1_{56} \\ \omega^2_{64} \\ \omega^3_{45} \end{pmatrix}, \qquad \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} = \begin{pmatrix} -\omega^1_{23} & \omega^4_{53} & \omega^4_{26} \\ \omega^5_{34} & -\omega^2_{31} & \omega^5_{61} \\ \omega^6_{42} & \omega^6_{15} & -\omega^3_{12} \end{pmatrix}. \tag{2.22}$$

The Jacobi identities imply the twelve constraints

$$\begin{aligned} b_{ij}a_j + b_{jj}a_i &= 0\,, \qquad i \neq j \\ b_{ik}b_{kj} + b_{kk}b_{ij} &= 0\,, \qquad i \neq j \neq k\,. \end{aligned} \tag{2.23}$$

There are some obvious solutions of these constraints. For instance, (1): $b_{ij} = 0$, $\forall i,j$; (2): $a_i = 0$, $b_{ij} = b_i\delta_{ij}$; (3): $a_i = a$, $b_{ij} = b$, $i \neq j$, $b_{ii} = -b$.

It is also enlightening to see how the twisted torus structure arises by T-dualizing a string background including constant NS-NS 3-form flux [22, 23]. To simplify the discussion, as internal space we take the flat torus $\mathrm{T}^6$, i.e. we neglect the warp factors needed to have a solution of the equations of motion. Concretely, we start from a type IIB background

$$\begin{aligned} ds^2 &= (dx^1)^2 + \cdots + (dx^6)^2\,, \\ \mathcal{H}_3 &= -a_1\, dx^1 \wedge dx^5 \wedge dx^6 - a_2\, dx^4 \wedge dx^2 \wedge dx^6 - a_3\, dx^4 \wedge dx^5 \wedge dx^3\,. \end{aligned} \tag{2.24}$$

We want to perform T-dualities in $x^1, x^2, x^3$. For the magnetic field we choose a gauge such that $\mathcal{B}_2$ does not depend on the dualized coordinates. We take

$$\mathcal{B}_2 = -a_1\, x^6\, dx^1 \wedge dx^5 - a_2\, x^4\, dx^2 \wedge dx^6 - a_3\, x^5\, dx^3 \wedge dx^4\,. \tag{2.25}$$

Using standard results (see e.g. [23, appendix A]) gives the transformed metric

$$\begin{aligned} ds'^2 = &(dx^1 + a_1 x^6 dx^5)^2 + (dx^2 + a_2 x^4 dx^6)^2 + (dx^3 + a_3 x^5 dx^4)^2 + \\ &+ (dx^4)^2 + (dx^5)^2 + (dx^6)^2 \,. \end{aligned} \tag{2.26}$$

Moreover, $\mathcal{H}_3' = 0$. All of the NS-NS flux is traded for metric flux. From the new metric we read off the following tangent 1-forms

$$\begin{aligned} \eta^1 &= dx^1 + a_1 x^6 dx^5 \,, \qquad & \eta^4 &= dx^4 \,, \\ \eta^2 &= dx^2 + a_2 x^4 dx^6 \,, \qquad & \eta^5 &= dx^5 \,, \\ \eta^3 &= dx^3 + a_3 x^5 dx^4 \,, \qquad & \eta^6 &= dx^6 \,. \end{aligned} \tag{2.27}$$

Taking the exterior derivatives we easily identify the structure constants: $\omega^1_{56} = a_1$, $\omega^2_{64} = a_2$, and $\omega^3_{45} = a_3$.

Finally, an important point is that metric fluxes are also quantized. For the $a_i$ we have just seen that they are obtained from T-duality of NS fluxes. In general this is needed for consistency of the twisted torus structure [42].

## 3. IIA superpotential and RR tadpoles due to general fluxes

It is instructive to check how a number of terms in the IIA superpotential (including some induced by metric fluxes) may be obtained applying T-duality transformations to the known type IIB results. Our starting point is the type IIB orientifold $\mathrm{T}^6/\Omega(-1)^{F_L} I_6$, where $I_6$ reflects the six internal coordinates $x^M$. There are 64 O3-planes whose charge can be cancelled by adding D3-branes and/or flux. To go to type IIA we will implement mirror symmetry which is the same as T-duality in $x^1, x^2, x^3$. In the type IIA picture there are then O6-planes, wrapping the $x^i$, $i = 1, 2, 3$, and one can add intersecting D6-branes.

We consider a factorized geometry in which $\Omega$ is given in (2.5). In order to generate a superpotential for the $\tau_i$ and the axion-dilaton we turn on NS $\overline{\mathcal{H}}_3$ and RR $\overline{\mathcal{F}}_3$ 3-form fluxes that are conveniently expanded in the basis (2.6). For $\overline{\mathcal{F}}_3$ we take the most general combination

$$\overline{\mathcal{F}}_3 = -m\alpha_0 - e_0\beta_0 + \sum_{i=1}^{3}(e_i\alpha_i - q_i\beta_i) \,. \tag{3.1}$$

Mirror symmetry transforms $\overline{\mathcal{F}}_3$ into RR fluxes $(\overline{F}_0, \overline{F}_2, \overline{F}_4, \overline{F}_6)$ in type IIA. For $\overline{\mathcal{H}}_3$ we instead restrict to

$$\overline{\mathcal{H}}_3 = h_0\beta_0 - \sum_{i=1}^{3} a_i\alpha_i \,. \tag{3.2}$$

Under the three T-dualities only $h_0$ remains as NS flux, i.e. $\overline{\mathcal{H}}_3 \to \overline{H}_3 = h_0\beta_0$. As discussed before, the $a_i$ become instead metric fluxes, in fact $\omega^1_{56} = a_1$, $\omega^2_{64} = a_2$ and $\omega^3_{45} = a_3$. We do not turn on $\overline{\mathcal{H}}_3 \sim \alpha_0$ because then $\mathcal{B}_2$ would depend on the $x^i$. We do not consider $\overline{\mathcal{H}}_3 \sim \beta_i$ fluxes either because they lead to more complicated metrics [23, 43, 29].

The type IIB superpotential induced by the fluxes is given by [1]

$$\mathcal{W} = \int (\overline{\mathcal{F}}_3 - \tau\overline{\mathcal{H}}_3) \wedge \Omega \,. \tag{3.3}$$

Here $\tau = C_0 + ie^{-\phi}$, where $C_0$ is the RR 0-form. Substituting (2.5), (3.1) and (3.2) we find

$$\mathcal{W} = e_0 + h_0\tau + i\sum_{i=1}^{3}(e_i + a_i\tau)\tau_i - q_1\tau_2\tau_3 - q_2\tau_1\tau_3 - q_3\tau_1\tau_2 + im\tau_1\tau_2\tau_3\,. \tag{3.4}$$

Upon mirror symmetry the $\tau_i$ go into Kähler moduli and $\tau$ becomes the IIA dilaton, $\tau \to iS$. Hence, we obtain the IIA superpotential

$$W_{ST} = e_0 + ih_0S + \sum_{i=1}^{3}(ie_i - a_iS)T_i - q_1T_2T_3 - q_2T_1T_3 - q_3T_1T_2 + imT_1T_2T_3\,. \tag{3.5}$$

Notice that for $a_i = 0$ this coincides with (2.14), with $h_i = 0$, that was derived following the analysis of [19]. For $a_i \neq 0$ it agrees with results of [17, 20].

In type IIB the fluxes contribute to the $C_4$ tadpole with coefficient

$$N_{\text{flux}} = \int \overline{\mathcal{H}}_3 \wedge \overline{\mathcal{F}}_3 = h_0m + a_1q_1 + a_2q_2 + a_3q_3\,, \tag{3.6}$$

where we already substituted the fluxes at hand. Under mirror symmetry $N_{\text{flux}}$ transforms into a $C_7$ tadpole in the direction of the O6-planes. This tadpole also receives contributions from D6-branes. In general we introduce piles of $N_a$ intersecting D6-branes wrapping the factorizable 3-cycle

$$\Pi_a = (n_a^1, m_a^1) \otimes (n_a^2, m_a^2) \otimes (n_a^3, m_a^3)\,, \tag{3.7}$$

and the corresponding orientifold images wrapping $\otimes_i(n_a^i, -m_a^i)$. Here $n_a^i$ ($m_a^i$) are the wrapping numbers along the $x^i$ ($y^i$) torus directions. Including the O6-planes, that wrap $\otimes_i(1,0)$, leads to the tadpole cancellation condition

$$\sum_a N_a n_a^1 n_a^2 n_a^3 + \frac{1}{2}(h_0m + a_1q_1 + a_2q_2 + a_3q_3) = 16\,. \tag{3.8}$$

This agrees with the result of [20]. Tadpoles due to fluxes of the NS and the RR 0-form have been considered in [44, 18, 21]. To our knowledge, tadpoles due to metric fluxes were first discussed in [7].

The tadpole condition can also be derived from the equation of motion for $C_7$ in type IIA. Let $G_2 = dC_1 + mB_2 + \overline{F}_2$ and ${}^*F_2 = F_8 = dC_7$, then the relevant piece in the action is

$$\int_{M_4\times Y}[C_7 \wedge dF_2 + C_7 \wedge (m\overline{H}_3 + d\overline{F}_2)] + \sum_a N_a \int_{M_4\times\Pi_a} C_7\,. \tag{3.9}$$

The first term arises from the kinetic energy $\int G_2 \wedge {}^*G_2$. The second term takes into account the coupling to D6-branes and O6-planes. The important point to notice is that $d\overline{F}_2 \neq 0$ due to the metric fluxes. For instance, using (2.21), we obtain $(d\overline{F}_2)_{456} = (a_1q_1 + a_2q_2 + a_3q_3)$, and thus recover (3.8). Moreover, from other components of $C_7$ there



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are further cancellation conditions

$$\begin{aligned}
&\sum_a N_a n_a^1 m_a^2 m_a^3 + \frac{1}{2}(mh_1 - q_1 b_{11} - q_2 b_{21} - q_3 b_{31}) = 0\,, \\
&\sum_a N_a m_a^1 n_a^2 m_a^3 + \frac{1}{2}(mh_2 - q_1 b_{12} - q_2 b_{22} - q_3 b_{32}) = 0\,, \\
&\sum_a N_a m_a^1 m_a^2 n_a^3 + \frac{1}{2}(mh_3 - q_1 b_{13} - q_2 b_{23} - q_3 b_{33}) = 0\,.
\end{aligned} \tag{3.10}$$

These also agree with the conditions found in [20].

We have just seen that the metric fluxes $b_{ij}$ create RR tadpoles. Recently it has been observed [17, 20] that they also generate superpotential terms involving the $U_k$, as expected from the fact that the fluxes $a_i$ produce terms involving $S$. Performing a generalized dimensional reduction on the twisted torus it has been shown in [20] that the superpotential for the complex structure moduli is an extension of $W_Q$ that can be expressed as

$$W_Q = \int_Y \Omega_c \wedge (\overline{H}_3 + dJ_c)\,. \tag{3.11}$$

Such an expression was already proposed in [22]. The metric fluxes appear in $dJ_c$ that is computed using (2.21). A similar modification of the superpotential occurs in heterotic compactifications on non-Kähler manifolds [45]. In our setup, computing $W_Q$ and combining with $W_K$ in (2.14) yields the full superpotential

$$\begin{aligned}
W = e_0 + ih_0 S + \sum_{i=1}^{3}\left[(ie_i - a_i S - b_{ii}U_i - \sum_{j\neq i} b_{ij}U_j)T_i - ih_i U_i\right] - \\
- q_1 T_2 T_3 - q_2 T_1 T_3 - q_3 T_1 T_2 + imT_1T_2T_3\,.
\end{aligned} \tag{3.12}$$

This is the result obtained in [20].

An obvious and interesting question is how the above superpotential changes when we include non-diagonal moduli related to parameters $\tau_{ia}$ and $t_{ia}$ that can appear in $\Omega$ and $J$. In (3.12) only the diagonal parameters, i.e. $\tau_i \equiv \tau_{i,i+3}$ and $t_i \equiv t_{i,i+3}$, are taken into account. Furthermore, we have only switched on diagonal fluxes, along (2.6) and (2.13), which we still continue to do. Now, from the general expressions of $W_K$ and $W_Q$, eqs. (2.10) and (3.11), it is easy to see that the diagonal fluxes $h_i$, $e_i$ and $a_i$, or $h_0$, do not excite the off-diagonal moduli, generically denoted $T'$ and $U'$. However, the fluxes $m$, $q_i$ and $b_{ij}$ can potentially generate a superpotential for $T'$ and $U'$ that is at least quadratic in these fields. The Kähler potential has also quadratic corrections. The upshot is that when we look for supersymmetric minima there is always a solution $T' = U' = 0$ and the diagonal moduli fixed as when $T'$ and $U'$ are not included. We know that at the point $T' = U' = 0$ there is a global $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry and furthermore, supersymmetry guarantees that this minimum is stable. In the following we will then disregard the off-diagonal moduli.

For future purposes we define

$$\widetilde{T}_I = (i, T_1, T_2, T_3)\,, \qquad A_{IJ} = \begin{pmatrix} -h_0 & h_1 & h_2 & h_3 \\ a_1 & b_{11} & b_{12} & b_{13} \\ a_2 & b_{21} & b_{22} & b_{23} \\ a_3 & b_{31} & b_{32} & b_{33} \end{pmatrix}$$

$$\widetilde{U}_I = (S, U_1, U_2, U_3)\,. \tag{3.13}$$

The $\widetilde{U}_I$ dependent superpotential, due to NS and metric fluxes, takes the simple form

$$W_Q = -\sum_{I,J=0}^{3} A_{IJ}\widetilde{T}_I\widetilde{U}_J\,. \tag{3.14}$$

The flux contribution to $C_7$ tadpoles can also be written in terms of the matrix $A$.

Recall that the metric fluxes are constrained by the Jacobi identities (2.23). For instance, there is a solution $b_{ji} = b_i$, $b_{ii} = -b_i$, $a_i = a$. Further choosing RR fluxes $q_i = -c_2$ and $e_i = c_1$, allows a configuration with $T_1 = T_2 = T_3 = T$. Then the superpotential reduces to

$$W = e_0 + 3ic_1T + 3c_2T^2 + imT^3 + ih_0S - 3aST - \sum_{k=0}^{3}(ih_k + b_kT)U_k\,, \tag{3.15}$$

If the fluxes $h_k$ and $b_k$ are independent of $k$, we can also set $U_1 = U_2 = U_3 = U$.

Given the fluxes leading to (3.15), the tadpole condition (3.8) becomes

$$\sum_a N_a n_a^1 n_a^2 n_a^3 + \frac{1}{2}(h_0m - 3ac_2) = 16\,. \tag{3.16}$$

On the other hand,

$$\begin{aligned} \sum_a N_a n_a^1 m_a^2 m_a^3 + \frac{1}{2}(h_1m + b_1c_2) &= 0\,, \\ \sum_a N_a m_a^1 n_a^2 m_a^3 + \frac{1}{2}(h_2m + b_2c_2) &= 0\,, \\ \sum_a N_a m_a^1 m_a^2 n_a^3 + \frac{1}{2}(h_3m + b_3c_2) &= 0\,. \end{aligned} \tag{3.17}$$

In the next section we will see that to obtain a minimum of the moduli scalar potential the fluxes must satisfy some relations that will in particular fix the sign of the flux contributions to the tadpoles.

## 4. Vacuum structure in IIA orientifolds with fluxes

In this section we analyze the moduli scalar potential induced by the fluxes. For the Kähler potential we assume the usual tree-level result displayed in (2.15). The scalar potential is then simply

$$V = e^K \left\{ \sum_{\Phi=S,T_i,U_i} (\Phi + \Phi^*)^2 |D_\Phi W|^2 - 3|W|^2 \right\}, \tag{4.1}$$

where $D_\Phi W = \partial_\Phi W + W \partial_\Phi K$ and $W$ is given in eq.(3.12). We want to look for solutions of $\partial V/\partial \Phi = 0$. A well known and easy to show general result is that the simpler unbroken susy conditions $D_\Phi W = 0$ imply a minimum.

As it happens in the IIB case [3, 10], it is quite complicated to perform a complete analysis of all possible minima induced by the flux superpotential. We have however explored several possibilities including $\mathcal{N}$=1 supersymmetric Minkowski vacua, no-scale models in Minkowski, $\mathcal{N}$=1 supersymmetric AdS vacua and non-supersymmetric AdS vacua. Before providing specific details let us make some general comments about these vacua and their comparison to IIB mirror orientifolds discussed in [3, 10]. In type IIB the flux-induced superpotential only depends on the axion-dilaton and complex structure fields, and all Kähler moduli remain undetermined. In the IIA case at hand all moduli appear in the superpotential and in principle may be fixed. There are however classes of type IIB minima whose mirror IIA dual is not included among our vacua. In particular, those arising when the IIB superpotential is linear in the axion-dilaton $\tau$ and at the same time quadratic/cubic in the complex structure fields. In fact, most of the examples in refs. [3, 10] belong to this class. A naive IIB/IIA mirror symmetry would suggest terms in the superpotential linear in $S$ and quadratic/cubic in the Kähler moduli which are not present in our superpotential (3.12). Otherwise, the IIA options for fluxes are richer and lead to many new possibilities, not only in the number of determined moduli but also in the contribution of fluxes to the different RR tadpoles, as we discuss below.

Constraining to the dependence on the 7 moduli, $S$ and the diagonal $U_i, T_i$, in general one finds that to get (non-runaway) minima the superpotential has to depend at least on four moduli. Note also that the fields $S$ and $U_i$ appear in a similar (linear) way in the superpotential so that, e.g. given a superpotential depending on $S$, one can obtain another model replacing $S \to U_i$ and properly relabelling the fluxes. Compared to the original model, the new one so derived has in general different contributions to the RR tadpoles. The same is true for the $T_i$ moduli in the $m = 0$ case. One can obtain new models replacing $T_i \to U_i$ and exchanging appropriately the fluxes. This will be illustrated below.

In this section we then use the flux-induced superpotential to study $\mathcal{N}$=1 and $\mathcal{N}$=0 type IIA vacua. In appendix A we will also discuss how $\mathcal{N}$=1 vacua in the presence of metric fluxes can be analyzed in terms of compactifications on the twisted torus seen as a manifold with SU(3) structure.

Let us now discuss the different types of vacua in turn. To describe the results we will denote $\mathrm{Re}T_i \equiv t_i$, $\mathrm{Im}T_i \equiv v_i$, $\mathrm{Re}S \equiv s$ and $\mathrm{Re}U_i \equiv u_i$.

### 4.1 Supersymmetric Minkowski vacua

Supersymmetric Minkowski vacua have $\langle D_\Phi W\rangle = 0$ with $\langle V\rangle = 0$ so that the cosmological constant vanishes. Clearly, (4.1) then implies $\langle W\rangle = 0$ and the supersymmetry condition reduces to $\langle \partial W/\partial \Phi\rangle = 0$. To simplify notation, in the following we stop writing vevs explicitly, it should be obvious when quantities are meant to be evaluated at the extremum.



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When $W$ depends on only three fields it suffices to analyze $W = W_K(T_1, T_2, T_3)$, purely due to RR fluxes. Other cases can be easily translated. The general solution of $D_{T_i} W_K = 0$ was found in [46] in relation to BPS black holes, and applied in [47] in the context of type IIA vacua with fluxes. If $W_K = 0$ there are only pathological solutions in which e.g. $t_1 = 0$.

Going to superpotentials depending on four moduli or more it is easy to find models with $\mathcal{N}$=1 supersymmetry in Minkowski space. One can obtain examples of this kind of minima with the superpotential depending on a) one $S/U_i$ modulus and three $T_i$; b) two $S/U_i$ fields and two $T_i$ and c) two $S/U_i$ and three $T_i$. We describe these cases below. We have not found any models with superpotentials depending on more than 5 fields. In all the examples we find we need the presence of metric fluxes and in addition $m = 0$. In all cases there is only a partial fixing of moduli, our examples fixing at most 3 complex linear combinations of moduli. The fluxes in these examples contribute to the RR tadpoles with the same sign as D6-branes do. However they may contribute in any of the four RR tadpole directions, depending on each case. This is explained below.

#### 4.1.1 Superpotentials depending on four moduli

With four fields it is enough to study in detail $W = W(S, T_1, T_2, T_3)$, c.f. (3.5), independent of the $U_i$. Other cases, e.g. $W(S, U_1, T_2, T_3)$, can be mapped into this by renaming fields and choosing parameters appropriately. We look for solutions of $W = 0$, $\partial_S W = 0$ and $\partial_{T_i} W = 0$. If there are no metric fluxes we find that $W$ must vanish identically to avoid $t_i = 0$. If instead $a_i \neq 0$ there are solutions provided $m = 0$. Moreover, taking the real part of $\partial_\Phi W = 0$ gives four homogeneous equations $a_1 t_1 + a_2 t_2 + a_3 t_3 = 0$ and $a_i s + q_j t_k + q_k t_j = 0$, $i \neq j \neq k$. To have $s$, $t_i \neq 0$ the determinant of this system must vanish, this is

$$(a_1 q_1 + a_2 q_2 - a_3 q_3)^2 = 4 a_1 q_1 a_2 q_2 \,. \tag{4.2}$$

Let us discuss now some particular cases.

**(i) Example SM-1.** We can for example take $a_1 = 0$ which requires $q_1 = 0$ to avoid $t_2,\ t_3 = 0$. The superpotential has the general form

$$W = e_0 + i h_0 S + i \sum_i e_i T_i - S(a_2 T_2 + a_3 T_3) - T_1(q_2 T_3 + q_3 T_2) \,. \tag{4.3}$$

It must be $a_2 q_2 = a_3 q_3$ and for consistency one also needs $e_2 a_3 = e_3 a_2$, $h_0 q_2 = a_3 e_1$, and $h_0 e_2 = e_0 a_2$. Note that these conditions are easily satisfied, e.g. for $h_0 = e_I = 0$. Neither the imaginary nor the real parts of the moduli are fully determined, only

$$h_0 = a_2 v_2 + a_3 v_3 \,, \qquad e_2 = a_2 \mathrm{Im} S + q_3 v_1 \,, \qquad t_3 = -\frac{q_3 t_2}{q_2} \,, \qquad s = -\frac{q_3 t_1}{a_2} \,. \tag{4.4}$$

For $s, t_i > 0$ we must have $q_2 q_3 < 0$ and $a_2 q_2 > 0$. Now, the flux term in the (3.8) RR tadpole is equal to $2 a_2 q_2$ and is therefore positive.

**(ii) Example SM-2.** The above example with $a_1 = 0$ can be used to analyze vacua in which we replace one Kähler, say $T_1$, by a complex structure modulus, say $U_1$. The $W(S, U_1, T_2, T_3)$ superpotential is now

$$W = -T_2(a_2S + b_{21}U_1) - T_3(a_3S + b_{31}U_1) + e_0 + ih_0S \; - ih_1U_1 + ie_2T_2 + ie_3T_3 \,. \quad (4.5)$$

This is clearly equivalent to (4.3) after renaming $T_1 \to U_1$, $e_1 \to -h_1$, $q_2 \to b_{31}$ and $q_3 \to b_{21}$. The physics is however different. In particular, since all $q_i = 0$ and $m = 0$, these fluxes do not contribute at all to the RR tadpoles. Thus, this is an example in which one can fix moduli without affecting tadpoles, the fluxes are essentially unconstrained, except from the consistency conditions mentioned above. We will see later that there are $\mathcal{N}$=1 supersymmetric AdS vacua in which one can fix all moduli without any restriction from RR tadpoles.

**(iii) Example SM-3.** Another simple solution of (4.2) is to take $a_2 = a_3 = -a_1/2$, and $q_2 = q_3 = -q_1/2$. Further selecting $h_0 = 0$ and $e_I = 0$ one has a superpotential

$$W = -a_1ST_1 - q_1T_2T_3 + \frac{1}{2}(T_2 + T_3)(a_1S + q_1T_1) \,. \quad (4.6)$$

One obtains a supersymmetric Minkowski minimum with $T_1 = T_2 = T_3 = a_1S/q_1$. The fluxes contribute $(3a_1q_1/2) > 0$ to the (3.8) RR tadpoles.

### 4.1.2 Superpotentials depending on five or more moduli

With five fields there are solutions of $W = 0$ and $\partial W/\partial\Phi = 0$. This is our next example.

**(iv) Example SM-4.** Consider the superpotential

$$W = -(q_3T_2 + q_2T_3)T_1 - (b_{22}T_2 + b_{32}T_3)U_2 - (b_{23}T_2 + b_{33}T_3)U_3 \,. \quad (4.7)$$

Observe that the non-zero $b_{ij}$ trivially satisfy the Jacobi identities (2.23). If the fluxes satisfy $q_2b_{22} = q_3b_{32}$, $q_2b_{23} = q_3b_{33}$ and $q_2q_3 < 0$, there is a solution with $|q_2|t_3 = |q_3|t_2$. There is also a relation $-q_3t_1 = b_{22}u_2 + b_{23}u_3$. To have $t_1 > 0$ for any $u_2$, $u_3 > 0$, we need $q_2b_{22} > 0$ and $q_2b_{23} > 0$. Hence, the flux piece in (3.10) is negative (same as D6-branes). In this particular case fluxes contribute $2b_{22}q_2$ to the last two RR tadpoles in (3.10).

As we have said, for the given class of $W$'s, in which the metric fluxes must satisfy the Jacobi identities (2.23), there cannot be supersymmetric Minkowski solutions when $W$ depends on more than five fields. To see this, first observe that without loss of generality we can always take the three $T_i$ to be among the fields in $W$. Next, from (3.14) we see that $\partial W/\partial\widetilde{U}_K = 0$ implies $A_{IJ}\widetilde{T}_I = 0$, and taking real part, $A_{iJ}t_i \equiv \mathcal{A}_{Ji}t_i = 0$, where $J$ takes three or four values. Note that the $t_i$ correspond to the kernel of the metric flux matrix $\mathcal{A}$. Thus to have solutions with $t_i \neq 0$, rank $\mathcal{A} \leq 2$. After using the Jacobi identities one is left with rank $\mathcal{A} = 1$. One can then check that the number of fields in $W$ can be at most five. However, we will see that for $W$ depending on all seven moduli, there are supersymmetric AdS minima in which $D_\Phi W = 0$ but $W \neq 0$.

As all examples so far show, fluxes allowing $\mathcal{N}$=1 supersymmetric Minkowski vacua only fix the moduli partially. Recall that in the type IIB case there are supersymmetric Minkowski minima with all complex structure fields fixed (but not the Kähler moduli). This is because, as already mentioned, in IIB there are extra superpotential couplings $\tau\tau_i\tau_j$ or $\tau\tau_1\tau_2\tau_3$, whose naive IIA mirror $ST_iT_j$, $ST_1T_2T_3$ we do not have. This situation changes when $W$ depends on the seven moduli. As we will see, supersymmetric AdS minima with all real moduli determined can then occur. This will be discussed in sections 4.3 and 4.4.

### 4.2 No-scale models in Minkowski space

As 'no-scale' we distinguish models in which the superpotential is independent of three of the moduli, so that the form (2.15) of the Kähler potential guarantees a cancellation of the cosmological constant [48]. In fact, the scalar potential is positive definite and it is minimized with respect to all fields when $D_{\Phi'}W(\Phi') = 0$. Since in general $W \neq 0$, supersymmetry is broken by the F-terms of the fields not appearing in $W$.

One easily finds no-scale minima with superpotentials depending on the moduli sets $\{U_I, T_1, T_2, T_3\}$ or $\{U_I, U_j, T_k, T_\ell\}$, with $U_I = S, U_i$. There are no minima coming from superpotentials of the form $W(U_I, U_j, U_k, T_\ell)$ or $W(S, U_1, U_2, U_3)$. Unlike the previous case with $\mathcal{N}$=1 supersymmetry, one can find no-scale models with and without metric fluxes. In the latter case one necessarily has $m \neq 0$. As in the $\mathcal{N}$=1 supersymmetric examples, the moduli are typically only partially fixed and the fluxes contribute to RR tadpoles like D6-branes. In particular, if $W$ only depends on $S, T_i$, the fluxes only contribute to the (3.8) RR tadpoles. If it depends on one or two $U_i$'s, fluxes may contribute to other RR tadpoles, but always positively. We can consider a superpotential $W = W(S, T_1, T_2, T_3)$ as generic, since one can always replace $S$ or one or two $T_i$'s by $U_i$'s if appropriate fluxes are also exchanged.

Our task is then to look for solutions of $D_SW = 0$ and $D_{T_i}W = 0$, without imposing $W = 0$, starting with the $W(S, T_1, T_2, T_3)$ given in eq. (3.5). We remind the reader that this superpotential may be obtained performing T-dualities on the superpotential $\mathcal{W}$ of type IIB generated by certain $\overline{\mathcal{F}}_3$ and $\overline{\mathcal{H}}_3$ fluxes. Now, in the type IIB case, solving $\mathrm{D}_\tau\mathcal{W} = 0$, $\mathrm{D}_{\tau_i}\mathcal{W} = 0$, is equivalent to demanding that the flux $\overline{\mathcal{G}_3} = (\overline{\mathcal{F}}_3 - \tau\overline{\mathcal{H}}_3)$ be imaginary self dual (ISD) [2]. Indeed, e.g. the conditions (4.13), (4.14) and (4.15) below amount to the ISD requirement.

#### 4.2.1 Examples with no metric fluxes

**(i) Example NS-1.** In the case with no metric fluxes one can check that there are minima only if $m \neq 0$, $h_0 \neq 0$, and $\gamma_1 = \gamma_2 = \gamma_3 = 0$, where the $\gamma_i$ are the combination of RR fluxes

$$\gamma_i = me_i + q_jq_k\,, \qquad i \neq j \neq k\,. \tag{4.8}$$

The superpotential has the form

$$W = e_0 + ih_0S + i\sum_i e_iT_i - q_1T_2T_3 - q_2T_1T_3 - q_3T_1T_2 + imT_1T_2T_3\,. \tag{4.9}$$

For the moduli we determine $v_i = -q_i/m$, $\mathrm{Im}S = (e_0m^2 - q_1q_2q_3)/h_0m^2$, whereas for the real parts $h_0s = mt_1t_2t_3$. Hence, $h_0m > 0$ and again the flux contribution to tadpoles is positive. We also find that at the minimum the superpotential is $W_0 = 2ih_0s$ and the gravitino mass is $m^2_{3/2} = h_0m/32u_1u_2u_3$. Note that this class of background is mirror to an analogous class of no-scale models in type IIB discussed in [11].

### 4.2.2 Examples with metric fluxes

With a superpotential of type $W(S,T_1,T_2,T_3)$ this amounts to having $a_i \neq 0$ for some $i$. Let us consider some simple models giving rise to a no-scale structure.

**(ii) Example NS-2.** Let us study first the case in which $m = 0$ and one of the $a_i$ vanishes, say $a_1 = 0$. We will then be able to translate the results for, say $W(S,U_1,T_2,T_3)$ if we wish. The superpotential has the form

$$W = e_0 + ih_0 S + i\sum_i e_i T_i - S(a_2T_2 + a_3T_3) - q_1T_2T_3 - q_2T_1T_3 - q_3T_1T_2\,. \qquad (4.10)$$

If $m = 0$, then $t_2, t_3 \neq 0$ require $q_1 = 0$. One finds that the axions are then fully determined to be

$$\mathrm{Im}S = \frac{e_2q_2 - e_3q_3}{q_2a_2 - q_3a_3}\,; \quad v_1 = \frac{e_3a_2 - e_2a_3}{q_2a_2 - q_3a_3}\,; \quad v_2 = \frac{h_0q_2 - e_1a_3}{q_2a_2 - q_3a_3}\,; \quad v_3 = \frac{e_1a_3 - h_0q_3}{q_2a_2 - q_3a_3}\,. \qquad (4.11)$$

There is a further relation $e_0 = h_0\mathrm{Im}S + e_1v_1$. The real parts verify $q_2t_1t_3 = a_2st_2$ and $q_3t_1t_2 = a_3st_3$. Thus we must have $a_2q_2 > 0$ and $a_3q_3 > 0$, indicating a positive contribution to the (3.8) RR tadpoles. On the other hand only pairwise ratios of real moduli are fixed, namely

$$s^2 = \frac{q_2q_3}{a_2a_3}t_1^2\,, \qquad t_3^2 = \frac{a_2q_3}{a_3q_2}t_2^2\,. \qquad (4.12)$$

At the minimum, $W_0 = -2s(a_2t_2 + a_3t_3)$.

In a variant of this model one can further set $a_2 = 0$ and for consistency $q_2 = 0$. The imaginary parts are obtained substituting these values in (4.11). For the real parts it only follows that $a_3st_3 = q_3t_1t_2$. In yet another variant one can set $a_2q_2 = a_3q_3$. The imaginary parts are then given as in (4.4), while $a_3t_3 = \pm a_2t_2$ and $a_2s = \pm q_3t_1$. This allows either $a_2a_3 > 0$ or $a_2a_3 < 0$ (so we could further impose $W_0 = 0$ as in the model SM-1).

**(iii) Example NS-3.** Let us now consider the case with $m \neq 0$, still with $a_1 = 0$. One finds that in order to have a solution the fluxes must verify

$$\gamma_2 = \frac{a_2\gamma_3}{a_3}\,, \quad h_0\gamma_3 = a_3(e_1q_1 + me_0)\,. \qquad (4.13)$$

For the imaginary parts we obtain

$$\mathrm{Im}S = \frac{me_0 + q_1e_1}{mh_0}\,; \quad v_1 = -\frac{q_1}{m}\,; \quad v_2 = -\frac{q_2}{m} + \frac{a_2st_2}{mt_1t_3}\,; \quad v_3 = -\frac{q_3}{m} + \frac{a_3st_3}{mt_1t_2}\,. \qquad (4.14)$$

The real parts instead satisfy

$$a_2a_3s^2 = \gamma_1t_1^2\,, \quad m^2t_1^2t_2^2t_3^2 + a_2^2s^2t_2^2 + a_3^2s^2t_3^2 = (h_0m + a_2q_2 + a_3q_3)st_1t_2t_3\,. \qquad (4.15)$$

This shows that $(h_0m + a_2q_2 + a_3q_3) > 0$ and hence the flux contribution to tadpoles is again positive. Notice that the above solution simplifies upon taking $a_2 = 0$ which is consistent if $\gamma_1 = \gamma_2 = 0$. In this case

$$t_3 = \frac{(h_0m + q_3a_3)st_1t_2}{(a_3s)^2 + (mt_1t_2)^2}\,. \qquad (4.16)$$

We also find that at the mimimum

$$W_0 = -\frac{2(h_0 m + q_3 a_3) s t_1 t_2}{a_3 s + i m t_1 t_2}\,. \tag{4.17}$$

The gravitino mass turns out to be

$$m_{3/2}^2 = \frac{(h_0 m + q_3 a_3)}{32 u_1 u_2 u_3}\,. \tag{4.18}$$

As expected for a no-scale model it only depends on the $u_i$.

**(iv) Example NS-4.** Consider now an isotropic setup with $a_1 = a_2 = a_3 = a$ and $q_1 = q_2 = q_3 = q$. It is possible to obtain vacua with the four moduli fixed [17]. At the minimum, $\mathrm{Re}T_k = t$, $\mathrm{Im}T_k = v$. After taking $e_I = 0$ we find the equations

$$\begin{aligned} v\left[2amv^2 - (h_0 m - 3aq)v - 2h_0 q\right] &= 0\,, \\ (h_0 - 3av)(q + mv) &= amt^2\,. \end{aligned} \tag{4.19}$$

The dilaton is given by $aS = (q+mv)T^* - iqv$. There is a solution with $v = 0$ which, since then $T = \sqrt{h_0 q/ma}$, $aS = qT$, can occur only if $h_0 m > 0$ and $aq > 0$. A solution with $v \neq 0$ can happen if $(aq/h_0 m) < -1$ and $h_0 m < 0$, or if $-1/9 < (aq/h_0 m) < 0$ and $h_0 m > 0$. In all cases the flux contribution to the RR tadpole is positive.

### 4.3 AdS vacua without metric fluxes

We now consider the superpotential, depending on all seven moduli, given by the sum of the two terms in (2.14). As we will see, as long as $m \neq 0$, $T_i$, $s$ and $u_i$ can be fixed in AdS, but only a linear combination of the axions $\mathrm{Im}S$ and $\mathrm{Im}U_i$ is determined. This can occur both for broken and unbroken supersymmetry but in all cases we prove that the contribution of fluxes to RR tadpoles is always positive.

We find that in the absence of metric fluxes the vacuum structure is determined by the combination of fluxes $\gamma_i$, c.f. (4.8). In particular, $\gamma_i < 0$ is required to solve $D_{T_i}W = 0$, $D_S W = 0$, and $D_{U_i} W = 0$ in the supersymmetric case. Then, the Kähler axions are fixed as $v_i = -q_i/m$, whereas for the other axions

$$h_0 \mathrm{Im}S - \sum_i h_i \mathrm{Im}U_i = \frac{1}{m^2}\left[e_0 m^2 - q_1 q_2 q_3 + \sum_i q_i \gamma_i\right]. \tag{4.20}$$

The real parts are instead determined to be

$$t_1 = \sqrt{\frac{5|\gamma_2\gamma_3|}{3m^2|\gamma_1|}}\,,\quad t_2 = \frac{\gamma_1 t_1}{\gamma_2}\,,\quad t_3 = \frac{\gamma_1 t_1}{\gamma_3}\quad ;\quad s = -\frac{2\gamma_1 t_1}{3mh_0}\,,\quad u_i = \frac{2\gamma_1 t_1}{3mh_i}\,. \tag{4.21}$$

Observe that in order to have $s > 0$, $u_k > 0$, it must be that $mh_0 > 0$ whereas $mh_k < 0$. Hence, the flux contribution to the tadpoles is positive in (3.8) and negative in (3.10). Note that in the present case only $m$, $h_0$ and $h_k$ are restricted by RR tadpole conditions while the fluxes $e_0$, $c_1$ and $c_2$ are essentially unconstrained. This will allow us to find minima at arbitrarily large volume and small dilaton, see below. Type IIA supersymmetric AdS vacua without metric fluxes have been recently addressed in [21]. We obtain similar results.

To go beyond supersymmetric minima and find all solutions of $\partial V/\partial\Phi = 0$ we will analyze the case $T_1 = T_2 = T_3 = T$, so that $W$ is given in (3.15) with $b_k = a = 0$. The vacuum structure now depends on $\gamma = mc_1 + c_2^2$. In particular, there exists a supersymmetric AdS minimum only if $\gamma < 0$. We also find that necessarily $mh_0 > 0$ and $mh_k < 0$. Therefore, the flux contribution to the tadpoles is positive in (3.16) and negative in (3.17).

To summarize the results we use the shorthand $\mathrm{Re}T = t$ and $\mathrm{Im}T = v$. The extremum only fixes one linear combination of the imaginary parts of dilaton and complex structure fields which is given by

$$h_0 \mathrm{Im}S - \sum_{k=1}^{3} h_k \mathrm{Im}U_k = e_0 - 3c_1 v - 3c_2 v^2 + mv^3 \,. \tag{4.22}$$

There are two branches for $v$, namely,

$$v_s = \frac{c_2}{m}\,, \qquad v_{ns} = \frac{c_2 \pm \sqrt{\gamma - m^2t^2/2}}{m}\,. \tag{4.23}$$

For each value of $v$ there are various sub-branches according to the relation among the real parts of $S$ and the $U_k$. From now on we just look at solutions with $h_1u_1 = h_2u_2 = h_3u_3$. In this case there are two sub-branches characterized by

$$\begin{aligned} (\mathrm{I}) &: h_k u_k = -h_0 s\,, \qquad k = 1,2,3 \\ (\mathrm{II}) &: h_k u_k = h_0 s - mt^3\,. \end{aligned} \tag{4.24}$$

In the $v_s$ sub-branch I,

$$m^2t^2 = \pm\frac{5}{3}\gamma\,, \qquad h_0 s = \frac{2}{5}mt^3\,, \qquad \Lambda_s = -\frac{\gamma^2}{24m^2 s u_1 u_2 u_3 t}\,. \tag{4.25}$$

For $\gamma < 0$ this is the AdS supersymmetric minimum. For $\gamma > 0$ it is a non-supersymmetric AdS extremum with same data for the moduli and the cosmological constant. In the $v_s$ sub-branch II,

$$m^2t^2 = \pm\frac{5}{\sqrt{6}}\gamma\,, \qquad h_0 s = \frac{4}{5}mt^3\,, \qquad \frac{\Lambda}{\Lambda_s} = \frac{32}{27}\left(\frac{6}{9}\right)^{1/4} \sim 1.071\,. \tag{4.26}$$

Both $\gamma < 0$ and $\gamma > 0$ are allowed. In either case it is a non-supersymmetric AdS extremum.

The $v_{ns}$ branch can occur only if $\gamma > 0$. There are two non-supersymmetric AdS sub-branches according to (4.24). Their data are

$$\begin{aligned} (\mathrm{I}): m^2t^2 &= \frac{4}{3}\gamma\,, \quad h_0 s = \frac{2\gamma t}{3m}\,, \quad \frac{\Lambda}{\Lambda_s} = \frac{25\sqrt{5}}{48} \sim 1.165\,, \\ (\mathrm{II}): m^2t^2 &= \frac{196}{99}\gamma\,, \quad h_0 s = \frac{14\gamma t}{9m}\,, \quad \frac{\Lambda}{\Lambda_s} = \frac{11^4 5^2 3^2\sqrt{55}}{2^4 7^7\sqrt{3}} \sim 1.070\,. \end{aligned} \tag{4.27}$$

This ends our list of non-supersymmetric AdS vacua.

Note that in all these examples without metric fluxes the fixed moduli scale with respect to the RR 4-form and 2-form fluxes $c_1$ and $c_2$ as

$$t \simeq s^{1/3} \simeq u_k^{1/3} \simeq \gamma^{1/2} \simeq c_1^{1/2}\,, \quad c_2\,, \tag{4.28}$$

for large fluxes. Thus the compactification volume may be arbitrarily large for large $c_1$ and/or $c_2$. For large fluxes, the four- and ten-dimensional dilatons behave as

$$e^{\phi_4} \simeq c_1^{-3/2}\,, \qquad c_2^{-3}\,; \qquad e^{\phi} \simeq c_1^{-3/4}\,, \qquad c_2^{-3/2} \tag{4.29}$$

so that the vacua lie in a perturbative regime for sufficiently large RR 4-form and/or 2-form fluxes. Concerning the cosmological constant, one can check that for large fluxes $c_1$ and $c_2$ it scales as

$$V_0 \simeq -\gamma^{-9/2} \simeq -c_1^{-9/2}\,, \qquad -c_2^{-9}\,. \tag{4.30}$$

Thus, for large $c_1/c_2$ the c.c. goes with the string dilaton like $e^{6\phi}$. The density of RR fluxes is also suppressed. As pointed out in [21], to compute this density a factor of $g_s = e^{\phi}$ must be included. Then, the flux density of $\overline{F}_4$ ($\overline{F}_2$) behaves like $c_1^{-3/2}$ ($c_2^{-3}$) for large $c_1$ ($c_2$) fluxes.

### 4.4 Supersymmetric AdS vacua with metric fluxes

We finally treat the full superpotential given in (3.12). We will see that all real moduli may be fixed at the minimum. Concerning the imaginary parts, some linear combinations of Im$S$, Im$U_i$ remain massless but, as we will discuss in the next section, in the presence of D6-branes those massless fields are in fact necessary for certain (potentially anomalous) brane U(1) fields to get a Stückelberg mass. We will also see that to get these minima certain discrete relationships among the fluxes must be fulfilled.

There are two classes of models depending on whether $m = 0$ or not. In the former case one finds that fluxes in general contribute to all RR tadpole directions with a sign which is *opposite* to that of D6-branes. This is important since it offers an alternative to orientifold planes to cancel RR tadpoles. In the second case with $m \neq 0$ one finds the interesting result that, depending on different flux choices, always including metric fluxes, the sign of the contribution to RR tadpoles may be arbitrary and the net contribution may vanish. In the latter case one has a cancellation of a positive RR-NS contribution $h_0 m$ with a RR-metric flux contribution of type $a_i q_i$. This is interesting because in this class of backgrounds all real moduli are determined but the fluxes are unconstrained by RR tadpole cancellation conditions.

We will examine the case $T_k = T$ and look for supersymmetric minima for any $m$. From $D_{U_k}W = 0$ and $D_S W = 0$, with $W$ given in (3.15), we find

$$3as = b_k u_k\,. \tag{4.31}$$

Hence, $a$ and $b_k$ must be both non-zero and of the same sign. Moreover, there are consistency conditions

$$3h_k a + h_0 b_k = 0\,, \qquad k = 1, 2, 3\,. \tag{4.32}$$

Therefore, either both $h_0$ and $h_k$ vanish or both are non-zero and of opposite sign. These conditions do not involve the moduli so at most we will have five equations for six unknowns, i.e. we will have at least one flat direction for the supersymmetric minima. In fact, only a combination of complex structure axions is fixed as

$$3a\mathrm{Im}S + \sum_{k=1}^{3} b_k \mathrm{Im}U_k = 3c_1 + \frac{3c_2}{a}(3h_0 - 7av) - \frac{3m}{a}v(3h_0 - 8av)\,. \tag{4.33}$$

If $h_0, h_k \neq 0$ using (4.32) we can write the fixed axion combination as $h_0\mathrm{Im}S - \sum_k h_k \mathrm{Im}U_k$. We also find

$$as = 2t(c_2 - mv)\,. \tag{4.34}$$

Except for some axion directions, we have thus determined all the moduli in terms of $T$ which is found from the remaining equations. The solution depends on whether $m$ is different from zero or not. We now specify to these two possibilities.

**(i)** $m = 0$**. Examples AdS-1.** When $m = 0$ we find the simple results

$$v = \frac{h_0}{3a}\,, \qquad 9c_2 t^2 = e_0 - \frac{h_0 c_1}{a} - \frac{h_0^2 c_2}{3a^2}\,. \tag{4.35}$$

At the minimum, $W_0 = -12c_2 t^2$ and the cosmological constant turns out to be

$$V_0 = \Lambda = -\frac{ab_1b_2b_3}{128\, c_2^2\, t^3}\,, \tag{4.36}$$

where $t$ is given in (4.35). It is important to notice that (4.34) fixes $c_2 a > 0$ so that in the supersymmetric minima with $m = 0$ the metric fluxes give a negative contribution to the tadpole in (3.16). Similarly, $c_2 b_k > 0$ and the flux contribution to the tadpoles in (3.17) is positive. This is the result we advertised.

Let us now check whether we have enough freedom to locate all moduli at large volume and small dilaton so that one can trust the effective 4-dimensional field theory approximation being used. The fluxes of type $a, b_k$ and $c_2$ are constrained by the RR tadpole cancellation conditions and by the extra conditions (4.32). The values of $h_0$ and $h_k$ are constrained by the latter but in principle both may be large as long as $h_0/h_k = -3a/b_k$. On the other hand, the fluxes of the RR 6-form ($e_0$) and 4-form ($c_1$) are unconstrained and may be arbitrarily large. Note then that for large $e_0$ and $c_1$ the moduli fields behave all like

$$t \simeq s \simeq u_k \simeq e_0^{1/2}\,, \qquad c_1^{1/2}\,. \tag{4.37}$$

In order for our vacuum to remain in a perturbative regime we would like to have small values for the 4-dimensional coupling $e^{\phi_4}$ and the 10-dimensional string coupling $e^{\phi}$. They are found to be

$$\begin{aligned} e^{\phi_4} &= (su_1u_2u_3)^{-1/4} = t^{-1}\frac{(ab_1b_2b_3)^{1/4}}{2\cdot 3^{3/4}c_2}\,, \\ e^{\phi} &= e^{\phi_4}t^{3/2} = t^{1/2}\frac{(ab_1b_2b_3)^{1/4}}{2\cdot 3^{3/4}c_2}\,. \end{aligned} \tag{4.38}$$

We thus see that for large $t$ (which may be obtained e.g. with a large 6-form flux $e_0$) the 4-dimensional dilaton is small. However the string dilaton grows with $t$. Only by appropriately choosing the fluxes, i.e. with large $c_2$ one can perhaps maintain it under control. On the other hand such fluxes are in general very much constrained by the RR tadpole conditions so it seems difficult having small string dilaton and large volume at the same time. We will see however that in the case with $m \neq 0$ one can easily stabilize the moduli in the perturbative regime.

**(ii) $m \neq 0$. Examples AdS-2.** To deal with $m \neq 0$ it is convenient to introduce a new variable for Im$T$. If $h_0 \neq 0$ we use

$$v = (\lambda + \lambda_0)\frac{h_0}{3a}\,, \qquad \lambda_0 = \frac{3c_2 a}{m h_0}\,. \tag{4.39}$$

The value of $\lambda$ follows from the cubic equation

$$160\lambda^3 + 186(\lambda_0 - 1)\lambda^2 + 27(\lambda_0 - 1)^2\lambda + \lambda_0^2(\lambda_0 - 3) + \frac{27a^2}{m h_0^3}(e_0 a - c_1 h_0) = 0\,. \tag{4.40}$$

Clearly, we need a real solution for $\lambda$ and moreover, such that $\lambda(\lambda + \lambda_0 - 1) > 0$ because now $t$ is determined from

$$3a^2 t^2 = 5h_0^2 \lambda(\lambda + \lambda_0 - 1)\,. \tag{4.41}$$

Notice also that (4.34) takes the form $3a^2 s = -2h_0 m\lambda t$. For the cosmological constant we find

$$V_0 = -\frac{a b_1 b_2 b_3 \lambda_0^2(16\lambda + \lambda_0 - 1)}{1920\, c_2^2\, t^3 \lambda^3}\,, \tag{4.42}$$

where $\lambda$ is the appropriate solution of (4.40) and $t$ is given in (4.41).

There is a variety of cases depending on the values of the different fluxes. One of the interesting features when $m \neq 0$ is that the contribution to the RR tadpoles may have either sign and even vanish. In fact, the flux-induced tadpoles in (3.16) and (3.17) are respectively $\frac{1}{2}h_0 m(1-\lambda_0)$ and $\frac{1}{2}h_k m(1-\lambda_0)$. Thus, the flux tadpoles vanish at the special value $\lambda_0 = 1$. This is important, as we mentioned above, since it allows to fix the moduli without any constraint from RR tadpole cancellations.

To analyze the equations that determine $\lambda$ and $t$ we can proceed in various ways. We could choose for example $e_0$ and $c_1$ and study the allowed values of $\lambda_0$. For instance, with $e_0 = c_1 = 0$, we find that to have solutions for $\lambda$ with acceptable $t$ necessarily $\frac{1}{3} < \lambda_0 < 3$. We also need $mh_0 < 0$ and $mh_k > 0$ so that $s > 0$ and $u_k > 0$. The special value $\lambda_0 = 1$ at which tadpoles vanish is allowed and leads in turn to

$$t = \sqrt{\frac{5}{3}}\lambda\left|\frac{h_0}{a}\right|\,, \qquad s = -\frac{2mh_0\lambda}{3a^2}t\,, \qquad u_k = -\frac{2mh_0\lambda}{ab_k}t\,. \tag{4.43}$$

From the cubic equation we find the value $\lambda = (10)^{2/3}/20$.

As we advanced, with $m \neq 0$ one can locate the minima in perturbative regions. Consider for instance the case $e_0 = c_1 = 0$ and $\lambda_0 = 1$ so that the real moduli are given in (4.43). Note that one can have $h_0$, $h_k$ and $c_2$ arbitrarily large as long as $\lambda_0 = 1$ and eq. (4.32) is respected. Then one can check that

$$e^{\phi_4} \simeq h_0^{-2}, \qquad e^{\phi} \simeq h_0^{-1/2}, \tag{4.44}$$

so that for sufficiently large $h_0$ the minima will be perturbative. Note also that the NS flux density is diluted for large fluxes since it goes like $h_0/t^{3/2} \simeq h_0^{-1/2}$. Concerning the RR flux $\overline{F}_2$, its density also goes like $h_0^{-1/2}$ for large fluxes, taking the factor of $g_s$ into account [21]. The cosmological constant eq. (4.42) scales like

$$V_0 \simeq -h_0^{-5} \simeq -(e^{\phi})^{10} \tag{4.45}$$

and hence is substantially suppressed for large $h_0$. Similar results may be obtained for values of $\lambda_0$ sufficiently close to 1, which would allow for contributions to RR tadpoles with either sign and of arbitrary size. In section 6 we will consider this possibility to construct a semi-realistic intersecting D6-brane model with all diagonal closed string moduli stabilized.

Let us mention for completeness other solutions within this class of AdS minima with $m \neq 0$. We may start by choosing a preferred value for some of the moduli. For example, we can set $v = 0$, and $h_0 \mathrm{Im} S - \sum_k h_k \mathrm{Im} U_k = 0$. Then, necessarily $c_1 = -3h_0 c_2/a$ and, from the cubic equation with $\lambda = -\lambda_0$, $e_0 = 45 h_0 c_2^2/ma$. This is the solution found in [20].

Another way to proceed with the analysis is to fix $\lambda_0$. For example, we can take $c_2 = 0$ so that $\lambda_0 = 0$. Obviously, $t^2 > 0$ then requires either $\lambda < 0$ or $\lambda > 1$. If $\lambda < 0$, then $s > 0$, $u_k > 0$ demand $h_0 m > 0$ and $h_k m < 0$, thus the flux contribution to tadpoles is like that of D6-branes. It is more interesting to consider $\lambda > 1$ so that $h_0 m < 0$ and $h_k m > 0$. Furthermore, to satisfy (4.40), it must be $(e_0 a - c_1 h_0) > 0$. Were it not for the fact that the fluxes are integers, we could always find solutions for some chosen $\lambda$. But still there is room to adjust the fluxes. For example, for $\lambda = 3/2$ we just need $(e_0 a - c_1 h_0) = -6 h_0^3 m/a^2$ and this could be verified say for $e_0 = 0$, $c_1 = 24m$ and $h_0 = 2a$.

We can also set $h_0 = h_k = 0$, but to this end a different parametrization of $v$, amounting to $\lambda \to \lambda_0 \hat{\lambda}$, must be employed. Now the interesting case is $\hat{\lambda} < -1$ because $s > 0$, $u_k > 0$ require $c_2 a > 0$ and $c_2 b_k < 0$ so that the flux contribution to tadpoles could cancel that of D6-branes. Again we can choose some $\hat{\lambda}$ and find values of $e_0$ to satisfy the cubic equation for $v$. For instance, for $\hat{\lambda} = -3/2$ we need $e_0 m^2 = -161 c_2^3$. One can check however that in this and the previous solution it is again hard to achieve at the same time a large value for the volume and a small value for the 10-dimensional dilaton, the reason being that now the value of fluxes $h_0$, $c_2$ and $h_k$ will be constrained by RR tadpole cancellation conditions.

One can also easily find non-susy AdS vacua. We will just show a particularly simple example. In general, there are solutions in which (4.31) and (4.32) are still satisfied. To go further let us set $m = 0$. Then there are solutions with $as = 2c_2 t$ and $v = h_0/3a$, but with the novelty that

$$\pm 9 c_2 t^2 = e_0 - \frac{h_0 c_1}{a} - \frac{h_0^2 c_2}{3a^2}. \tag{4.46}$$

With plus sign this is the supersymmetric minimum, but we can also choose the minus sign depending on the fluxes. For instance, if $e_0 = c_1 = 0$, only the non-supersymmetric choice is available. In this case the minimum is AdS and it is typically stable because the eigenvalues of the Hessian are positive or negative but above the Breitenlohner-Freedman bound [49].

## 5. D6-branes, fluxes and the Freed-Witten anomaly

We are going to consider now adding D6-branes to a IIA background with fluxes turned on. Besides the general RR tadpole cancellation constraints already mentioned, a number of points do also change. One apparent puzzle is the following. In the world-volume of a generic stack of D6-branes there is a U(1) gauge field whose scalar partner parametrizes the D6-brane position in compact space. These U(1)'s often get Stückelberg masses by swallowing RR scalars and disappear from the low-energy spectrum. At the same time these scalars participate in the cancellation of U(1) gauge anomalies through a variation of the Green-Schwarz mechanism [50]. Let us specify now to the toroidal case considered in detail in the present paper. Consider a $\mathrm{D6}_a$ wrapping a factorizable torus with wrapping numbers as in eq. (3.7). Then the $\mathrm{U}(1)_a$ field couples to RR fields in $D$=4 as follows

$$F^a \wedge N_a \sum_{I=0}^{3} c_I^a C_I^{(2)} \tag{5.1}$$

with

$$c_0^a = m_a^1 m_a^2 m_a^3\,; \qquad c_1^a = m_a^1 n_a^2 n_a^3\,; \qquad c_2^a = n_a^1 m_a^2 n_a^3\,; \qquad c_3^a = n_a^1 n_a^2 m_a^3\,. \tag{5.2}$$

Here the $C_I^{(2)}$ are 2-forms which are Poincaré duals in $D$=4 to the $\mathrm{Im}U_I$ fields considered above. In terms of them the couplings have a Higgs-like form

$$A_\mu^a \partial^\mu (c_0^a\ \mathrm{Im}S - c_1^a \mathrm{Im}U_1 - c_2^a \mathrm{Im}U_2 - c_3^a \mathrm{Im}U_3)\,. \tag{5.3}$$

We thus observe that certain linear combinations of $\mathrm{Im}U_I$ ($U_0 = S$) fields get a mass by combining with open string vector bosons living on the branes. Moreover, these linear combinations of $\mathrm{Im}U_I$ fields will transform with a shift under $\mathrm{U}(1)_a$ gauge transformations, like Goldstone bosons do. On the other hand, we have seen above that NS and metric backgrounds give rise to terms in the superpotential linear in $\mathrm{Im}U_I$, i.e.

$$W_Q = \int_Y \Omega_c \wedge (\overline{H}_3\ + dJ_c) = - \sum_{I,J=0}^{3} A_{IJ} \widetilde{T}_I \widetilde{U}_J\,. \tag{5.4}$$

Such terms generically may give rise to potential terms for the $\mathrm{Im}U_I$ fields which would not be invariant under the shifts induced by $\mathrm{U}(1)_a$ gauge transformations. The condition to restore consistency and gauge invariance would be to impose the constraint

$$\int_{\Pi_a} (\overline{H}_3\ + \omega J_c) = 0\,, \tag{5.5}$$

evaluated at the appropriate vacuum. Here $\Pi_a$ denotes the 3-cycle wrapped by the D6-brane, $\omega$ are the metric fluxes and $J_c$ is the complexified Kähler 2-form of the torus. We have used $dJ_c = \omega J_c$, according to (2.21).

In absence of metric fluxes eq. (5.5) may be understood in terms of the Freed-Witten anomaly [33, 34, 7].[1] A simple way to see the origin of this constraint starting from type IIB is as follows [7]. Consider a D3-brane wrapping a 3-cycle $\Pi$ through which there is

[1] We thank A. Uranga for pointing out this connection to us.

some quantized NS flux $\overline{H}_3$. On the world-volume of the D3-brane there is a CS coupling of the form $\int C_2 \wedge F_2$, $C_2$ being the RR 2-form and $F_2$ the open string gauge field strength. After performing a IIB S-duality transformation one gets a coupling

$$\int_{\Pi \times \mathbb{R}} H_3 \wedge \tilde{A}_1 \,, \tag{5.6}$$

where $\tilde{A}_1$ is the gauge field dual to the $A_1$ form living on the D3-brane. This shows that a background for $H_3$ gives rise to a tadpole for $\tilde{A}_1$ and hence to an inconsistency. Performing three T-dualities one expects for D6-branes the analogous term

$$\int_{D6} H_3 \wedge \tilde{A}_4 \,. \tag{5.7}$$

The resulting tadpole is avoided if $\int_{\Pi_a} \overline{H}_3 = 0$. Equation (5.5) should thus be the extension of this constraint to the case including metric fluxes $\omega \neq 0$.

Note that, ignoring for the moment the effect of metric fluxes, eq. (5.5) implies that all D6-branes wrapping the orientifold should obey

$$\sum_{I=0}^{3} c_I^a h_I = 0 \,. \tag{5.8}$$

This is in general a strong constraint on the possible D6-branes one may add in specific models with flux, as we discuss below in specific examples. Remarkably, this constraint guarantees that combinations of axions getting masses by mixing with vector bosons are orthogonal to those becoming massive from fluxes, the latter being typically of the form $h_0 \mathrm{Im} S - \sum_k h_k \mathrm{Im} U_k$.

Another interesting point is the following. We have seen in section 4.4 that adding fluxes one can fix the torus moduli in a supersymmetry-preserving AdS minimum. Now, for non-zero NS fluxes one finds at the minima that $h_i/h_0 = -s/u_i$. Substituting this in eq. (5.8) and multiplying by the torus volume one arrives at

$$m_a^1 m_a^2 m_a^3 (R_y^1 R_y^2 R_y^3) - m_a^1 n_a^2 n_a^3 (R_y^1 R_x^2 R_x^3) - n_a^1 m_a^2 n_a^3 (R_x^1 R_y^2 R_x^3) - n_a^1 n_a^2 m_a^3 (R_x^1 R_x^2 R_y^3) = 0 \,. \tag{5.9}$$

This condition means that the D6-brane wraps a special Lagrangian cycle (slag). From the effective Lagrangian point of view this is proportional to a Fayet-Iliopoulos (FI) term [38] and hence it imposes dynamically that the D6-brane configuration should be supersymmetric (i.e. all FI terms should vanish). Thus one concludes that, in this class of AdS supersymmetric minima the constraint (5.8) implies that the brane configuration should be also supersymmetric. Notice that including metric fluxes in this class of minima does not add extra constraints to be satisfied due to the relations (4.32).

The condition (5.9) in these AdS vacua is in fact

$$\mathrm{Im}\Omega|_{\Pi_a} = 0 \,. \tag{5.10}$$

It arises from $\int_{\Pi_a} \overline{H}_3 = 0$ because at the AdS minimum $\overline{H}_3 \propto \mathrm{Im}\Omega$. In turn, in our setup this is a simple consequence of $h_i/h_0 = -s/u_i$. This is also found in a more general analysis of type IIA susy AdS vacua [64, 65]. Likewise, if there are metric fluxes, $dJ_c \propto \mathrm{Im}\Omega$. For instance, in the models of section 4.4 this can be deduced using (4.31). Therefore, even if the NS fluxes vanish, in these models there is still a FW constraint of the form

$$3ac_0^a - b_1 c_1^a - b_2 c_2^a - b_3 c_3^a = 0\,. \tag{5.11}$$

This guarantees that combinations of axions acquiring a mass from fluxes or from U(1) mixing are orthogonal to each other

Recently models of type IIB orientifolds with fluxes and intersecting (or rather magnetized) D-branes with semi-realistic spectrum have been constructed [6, 7, 10 – 12]. Some of them do not verify the (IIB version of) constraint (5.8) and hence would be in principle inconsistent. This possible problem with the FW anomaly was already pointed out in [7] where it was suggested that it could be cured if additional D-branes were included. In the case of IIA orientifolds under consideration we would need to add D4-branes hanging between different sets of D6-branes and their orientifold mirrors. It may be argued [7] that the chiral spectra from intersecting D6-brane models does not get affected by the presence of these extra D4-branes. However, no specific construction with this possible cancellation mechanism has been presented in the literature. In addition, it is not clear whether in the case of supersymmetric D6-brane configurations the addition of the extra D-branes does not spoil supersymmetry. Given this fact, it seems sensible to impose the constraint (5.8) on specific models with Minkowski vacua and that will be our approach below. In AdS vacua, in which the real parts of all moduli are determined, the FW anomaly cancels automatically if the brane configuration is supersymmetric.

## 6. Intersecting D6-brane models in the presence of fluxes

We have shown in previous sections that the addition of fluxes in type IIA theory leads to new properties not present in analogous IIB models. Some of the aspects we have found with potential model-building applications are: 1) fluxes may contribute to all four RR tadpoles, 2) one can have examples of fluxes fixing part or all of closed string moduli but not contributing to RR tadpoles, and 3) there are models with metric fluxes (as well as other NS and RR fluxes) in which one can obtain AdS supersymmetric vacua with all moduli stabilized and contribution to RR tadpoles *opposite to that of D6-branes*. In addition to these properties, since plenty of flux variables do not contribute to RR tadpoles, there is substantial freedom in the choice of the parameters of the vacua and in particular one can obtain minima at large volume and small dilaton values, in which the approximations inherent to a 4-dimensional effective lagrangian approach hold.

To illustrate the possible applications to model-building of these results in previous sections we are going to consider here specific intersecting D6-branes models with semi-realistic spectrum. The first two examples correspond to Minkowski vacua both with unbroken $\mathcal{N}=1$ supersymmetry and with broken supersymmetry but no-scale structure. Although in these cases only some of the closed string moduli are fixed at the minima, it is interesting to consider them since other effects could perhaps stabilize the rest of the moduli. In these two cases the models will be left-right symmetric extensions of the

| $N_i$ | $(n_i^1, m_i^1)$ | $(n_i^2, m_i^2)$ | $(n_i^3, m_i^3)$ |
|---|---|---|---|
| $N_a = 8$ | $(1, 0)$ | $(3, 1)$ | $(3, -1)$ |
| $N_b = 2$ | $(0, 1)$ | $(1, 0)$ | $(0, -1)$ |
| $N_c = 2$ | $(0, 1)$ | $(0, -1)$ | $(1, 0)$ |
| $N_{h_1} = 2$ | $(-2, 1)$ | $(-3, 1)$ | $(-4, 1)$ |
| $N_{h_2} = 2$ | $(-2, 1)$ | $(-4, 1)$ | $(-3, 1)$ |
| $8N_f$ | $(1, 0)$ | $(1, 0)$ | $(1, 0)$ |

**Table 1:** Wrapping numbers giving rise to a MSSM-like spectrum. Branes $h_1$, $h_2$ and $o$ are added in order to cancel RR tadpoles.

MSSM, with gauge group $\mathrm{SU}(3) \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L} \times \mathrm{U}(1)$, rather than the MSSM. The third example has an AdS supersymmetric background and is particularly interesting since, to our knowledge, is the first semi-realistic three-generation model with all closed string moduli stabilized. In this case also the gauge group is closer to that of the MSSM, since it is that of the SM with some additional U(1)'s. We consider models with non-supersymmetric intersecting branes and all closed string moduli fixed in appendix B.

### 6.1 Minkowski MSSM-like

In the class of type IIB orientifold models with fluxes studied up to now, it has been shown that flux backgrounds with Minkowski geometry, either $\mathcal{N}$=1 supersymmetric or not, lead to positive contributions to RR tadpoles. This stems from the fact that ISD fluxes always contribute to RR tadpoles as D-branes do. In building semi-realistic models this leads to problems with RR tadpole cancellation conditions, since typically fluxes contribute too much to tadpoles. It was pointed out in [11] that this problem may be cured if appropriate additional D9-anti-D9-brane pairs contributing negatively to some of the RR tadpoles are added. In any case, full cancellation of RR tadpoles in realistic toroidal models require considering orbifold generalizations like $\mathbb{Z}_2 \times \mathbb{Z}_2$ [51, 52]. Semi-realistic $\mathcal{N}$=1 supersymmetric type IIB $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifolds with flux backgrounds have been studied in [6, 7, 10 − 12]. The class of models of [11] has a brane content as given in table 1. In the case of the IIB $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold the $(n, m)$ integers would be magnetic numbers whereas in the T-dual IIA orientifold they correspond to wrapping numbers along horizontal and vertical directions of each $\mathrm{T}^2$ in the factorized $\mathrm{T}^6$ respectively. Note that this set contains as a subset the MSSM-like model introduced in [53, 54]. We assume as in [54] that the $b$ and $c$ D6-branes sit on top of the orientifold plane so that the corresponding gauge symmetries are enhanced to $\mathrm{SU}(2)_L$ and $\mathrm{SU}(2)_R$ respectively. The full initial gauge group is then $\mathrm{U}(4) \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times [\mathrm{U}(1)_1 \times \mathrm{U}(1)_2]$. Separating one of the $a$-branes from the other three produces the breaking $\mathrm{U}(4) \to \mathrm{U}(3) \times \mathrm{U}(1)$. Furthermore, two out of the three U(1)'s get a Stückelberg mass by combining with RR axion fields. We are thus left with a gauge group $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L} \times [\mathrm{U}(1)]$, which contains the left-right symmetric extension of the SM with an extra U(1). The branes $a, b, c$, give rise to a 3-generation MSSM-like spectrum whereas the additional branes $h_{1,2}$ in table 1 are used to help in cancelling the RR tadpoles.

Note that in the case of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ IIA orientifold the RR tadpole cancellation conditions in the presence of fluxes will have the form

$$\begin{aligned}
\sum_a N_a n_a^1 n_a^2 n_a^3 + \frac{1}{2}(h_0 m + a_1 q_1 + a_2 q_2 + a_3 q_3) &= 16\,, \\
\sum_a N_a n_a^1 m_a^2 m_a^3 + \frac{1}{2}(m h_1 - q_1 b_{11} - q_2 b_{21} - q_3 b_{31}) &= -16\,, \\
\sum_a N_a m_a^1 n_a^2 m_a^3 + \frac{1}{2}(m h_2 - q_1 b_{12} - q_2 b_{22} - q_3 b_{32}) &= -16\,, \\
\sum_a N_a m_a^1 m_a^2 n_a^3 + \frac{1}{2}(m h_3 - q_1 b_{13} - q_2 b_{23} - q_3 b_{33}) &= -16\,,
\end{aligned} \tag{6.1}$$

where the $(-16)$ in the last three conditions is the RR tadpole contribution of the other 3 orientifold planes existing in the $\mathbb{Z}_2 \times \mathbb{Z}_2$ case. Note that the branes $h_1$ and $h_2$ contribute negatively to all four RR tadpoles so that in principle one can use them to compensate for a too large contribution to the first tadpole condition from fluxes. Precisely this was the approach in ref.[11] (see also [55]). Here we will use this class of models as our starting point for the IIA orientifold case. Here are some possibilities:

**(i) A 3 generation $\mathcal{N}$=1 MSSM-like model with some fixed moduli.** Consider the above model in which we turn on non-vanishing fluxes as in one of the susy Minkowski examples of section 4.1 with non-vanishing $b_{31}, b_{21}, a_2, a_3$ (example *SM-2*). The addition of NS fluxes $h_0, h_1$ and RR $e_0, e_2, e_3$, is optional, but we set all the remaining backgrounds to zero. The superpotential has then the form

$$W = -T_2(a_2 S + b_{21} U_1) - T_3(a_3 S + b_{31} U_1) + e_0 + i h_0 S - i h_1 U_1 + i e_2 T_2 + i e_3 T_3\,. \tag{6.2}$$

As explained in section 4.1 this has a Minkowski supersymmetric minimum with

$$h_0 = a_2 v_2 + a_3 v_3\,, \qquad e_2 = a_2 \mathrm{Im} S + b_{21} \mathrm{Im} U_1\,, \qquad t_3 = -\frac{b_{21} t_2}{b_{31}}\,, \qquad s = -\frac{b_{21} u_1}{a_2}\,. \tag{6.3}$$

as long as $e_2 a_3 = e_3 a_2$, $h_0 b_{31} = -a_3 h_1$, $h_0 b_{21} = -a_2 h_1$ and $h_0 e_2 = e_0 a_2$. Thus in this supersymmetric Minkowski background two complex linear combinations of moduli are fixed at the minimum.

Note that, since $m = q_i = 0$, in this background the fluxes do not contribute to the RR tadpole. Thus one can consider the addition of D6-branes as in the the case with $N_f = 5$ in table 1. As pointed out in [11] with this choice all RR tadpoles cancel without the addition of fluxes in type IIB theory. In the present IIA case we can rather add the background considered here and the RR tadpoles are not modified and hence cancel. However the moduli are partially fixed by eq. (6.3).

It is easy to check that the $a, b$ and $c$ branes where the SM lives trivially satisfy the FW constraint. However the branes of type $h_{1,2}$ may be problematic unless:

$$a_2(m_a^1 m_a^2 m_a^3) - b_{21}(m_a^1 n_a^2 n_a^3) = a_2 - 12 b_{21} = 0 \tag{6.4}$$

which on the other hand may be easily satisfied by appropriately choosing $a_2$, $b_{21}$. Note that this condition guarantees that the linear combination of Im$S$, Im$U_1$ getting masses through fluxes (eq. (6.3)) is orthogonal to the linear combination getting masses by mixing with the U(1)'s of branes $h_{1,2}$.

Note that in the IIB version of this orientifold with fluxes considered in [11], the latter contributes to RR tadpoles and one can only get a one-generation $\mathcal{N}$=1 supersymmetric model.

**(ii) A 3 generation no-scale model.** One can also consider one of the no-scale backgrounds discussed in section (4.2), the variant of the *NS-2* model, and include a set of D6-branes as in table 1. A simple example is as follows. Take non-vanishing $a_3, q_3$ with the remaining $q_i = a_i = 0$. In addition one may include non-vanishing $h_0, e_0, e_i$ but set the remaining backgrounds to zero. The superpotential has then the form

$$W(S,T_i) = -a_3 S T_3 - q_3 T_1 T_2 + e_0 + i h_0 S + i \sum_i e_i T_i\,. \tag{6.5}$$

The imaginary part of $S$ and the $T_i$ are fixed as in eq. (4.11) whereas one has for the real parts the relationship $a_3 s t_3 = q_3 t_1 t_2$. In addition one has the constraint $e_0 = h_0 \mathrm{Im}S + e_1 v_1$. There is only a contribution equal to $\frac{1}{2} a_3 q_3$ to the first RR tadpole. We consider fluxes quantized in units of 8 to avoid problems with flux quantization [6, 7]. One can then cancel tadpoles in a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold with branes as in table 1 with $N_f = 1$ and $a_3 = q_3 = 8$.

One can also consider a no-scale model with a non-vanishing IIA mass parameter $m$ and with no metric fluxes, as described at the beginning of subsection 5.2. One takes non-vanishing $m$ and $h_0$. In addition one can have non-vanishing $e_i, q_j$, verifying $\gamma_i = m e_i + q_j q_k = 0$ $(i \neq j \neq k)$. Setting $h_0 = m = 8$ and $N_f = 1$ one cancels all tadpoles. Note that this model, which has no metric fluxes, is the IIA mirror of a similar no-scale model considered in [11].

One can check however that both these no-scale models as they stand have FW anomalies. The danger comes from the $h_{1,2}$ branes which have a non-vanishing product $m_a^1 m_a^2 m_a^3 \neq 0$. One possibility which might cure this problem is if, as suggested in [11], the brane $h_1$ recombines with the mirror of $h_2$ into a single (non-factorizable) D6-brane $h_1 + h_2'$. One can in fact claim that this is the generic situation for branes like these which do intersect. In this case, since $h_1$ and $h_2'$ have equal and opposite $m_a^1 m_a^2 m_a^3$, the FW would cancel on the recombined brane. On the other hand it is not clear whether after the addition of fluxes a flat direction in the effective potential exists corresponding to the recombination of those branes. In the $\mathcal{N}$=1 supersymmetric AdS model which we describe next no such problem appears.

### 6.2 A $\mathcal{N}$=1 MSSM-like model with all closed string moduli stabilized in AdS

The previous intersecting brane models were able to combine a semi-realistic spectrum with a partial determination of some closed string moduli. We now show that all such moduli may be stabilized in the case of AdS vacua, thus providing, to our knowledge, the first semi-realistic string model with all closed string moduli stabilized at weak coupling.

| $N_i$ | $(n_i^1, m_i^1)$ | $(n_i^2, m_i^2)$ | $(n_i^3, m_i^3)$ |
|---|---|---|---|
| $N_a = 4$ | $(1, 0)$ | $(3, 1)$ | $(3, -1)$ |
| $N_b = 1$ | $(0, 1)$ | $(1, 0)$ | $(0, -1)$ |
| $N_c = 1$ | $(0, 1)$ | $(0, -1)$ | $(1, 0)$ |
| $N_{h_1} = 3$ | $(2, 1)$ | $(1, 0)$ | $(2, -1)$ |
| $N_{h_2} = 3$ | $(2, 1)$ | $(2, -1)$ | $(1, 0)$ |
| $N_o = 4$ | $(1, 0)$ | $(1, 0)$ | $(1, 0)$ |

**Table 2:** A MSSM-like model with tadpoles cancelled by fluxes. Branes $h_1$, $h_2$ and $o$ are added in order to cancel RR tadpoles.

Note first that in the past it has been argued that it is impossible to construct semi-realistic $\mathcal{N}$=1 supersymmetric intersecting D6-brane models wrapping the IIA orientifold $T^6/(\Omega(-1)^{F_L} I_3)$. The reason for this was essentially the impossibility to cancel the 4 RR tadpole conditions simultaneously while maintaining supersymmetry. To obtain $\mathcal{N}$=1 supersymmetric models extra orbifold twisting (e.g. $\mathbb{Z}_2 \times \mathbb{Z}_2$, as in previous examples) had to be added, giving rise to extra orientifold planes which help in the cancellation of RR tadpoles [51]. We will show here that one can build $\mathcal{N}$=1 supersymmetric configurations in the purely toroidal orientifold in which those RR tadpoles may be cancelled by the addition of NS/RR and metric fluxes. The role played by additional orientifold planes in orbifold (e.g. $\mathbb{Z}_2 \times \mathbb{Z}_2$) models is here played by the additional fluxes which contribute like orientifold planes. At the same time those fluxes stabilize all closed string moduli in AdS space. Moreover the complex structure moduli are fixed at values which render the D6-brane configuration supersymmetric. Notice that in the $\mathcal{N}$=1 supersymmetric models previously considered in the literature those moduli where not determined by the dynamics.

Let us consider the set of D6-branes wrapping factorizable cycles in the orientifold as in table 2. Note that this set only differs from the previous examples in the form of the additional branes $h_1, h_2$. Another difference is that in our IIA case we have a purely toroidal (no $\mathbb{Z}_2 \times \mathbb{Z}_2$) orientifold without further twisting. The corresponding chiral spectrum at the intersections is given in table 3. In the table a prime indicates the mirror brane. The gauge group after separating branes and after two of U(1)'s get Stückelberg masses is $\mathrm{SU}(3) \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_R \times \mathrm{U}(1)_{B-L} \times [\mathrm{U}(1) \times \mathrm{SU}(3)^2]$. Note that, unlike the case of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ models above, one can make the breaking $\mathrm{SU}(2)_R \to \mathrm{U}(1)_R$ by brane splitting, and hence the gauge group is that of the MSSM supplemented by some extra U(1)'s. We have three generations of quarks and leptons, one Higgs multiplet $H$ and extra matter fields involving the auxiliary branes $h_1$, $h_2$ and $o$.[2]

With this brane content (plus the mirrors) the RR tadpole cancellation conditions are

$$\begin{aligned} 64 \ + \ \frac{1}{2}(h_0 m + a_1 q_1 + a_2 q_2 + a_3 q_3) \ &= \ 16\,, \\ -4 \ + \ \frac{1}{2}(h_1 m - q_1 b_{11} - q_2 b_{21} - q_3 b_{31}) \ &= \ 0\,, \end{aligned}$$

[2]One can check that if the branes $h_1$ and $h_2'$ recombine, most of the extra matter beyond the SM disappears from the massless spectrum, with only additional $\mathrm{SU}(2)_{L,R}$ doublets remaining.

| Intersection | Matter fields | Rep. | $Q_{3B+L}$ | $Q_1$ | $Q_2$ |
|---|---|---|---|---|---|
| $a-b$ | $F_L$ | $3(4,2_L)$ | 1 | 0 | 0 |
| $a-c$ | $F_R$ | $3(\bar{4},2_R)$ | -1 | 0 | 0 |
| $b-c$ | $H$ | $(2_L,2_R)$ | 0 | 0 | 0 |
| $a-h_1$ | $T_1$ | $(4,\bar{3}_1)$ | 1 | -1 | 0 |
| $a-h_1'$ | $T_1'$ | $5(4,3_1)$ | 1 | 1 | 0 |
| $a-h_2$ | $T_2$ | $5(\bar{4},3_2)$ | -1 | 0 | 1 |
| $a-h_2'$ | $T_2'$ | $(\bar{4},\bar{3}_2)$ | -1 | 0 | -1 |
| $b-h_2$ | $D_2$ | $2(2_L,\bar{3}_2)$ | 0 | 0 | -1 |
| $c-h_1$ | $D_1$ | $2(2_R,\bar{3}_1)$ | 0 | -1 | 0 |
| $h_1-h_2'$ | $X$ | $4(\bar{3}_1,\bar{3}_2)$ | 0 | -1 | 1 |

**Table 3:** Chiral spectrum of the MSSM-like model.

$$
\begin{aligned}
-4 \ &+ \ \frac{1}{2}(h_2 m - q_1 b_{12} - q_2 b_{22} - q_3 b_{32}) \ = \ 0 \, , \\
-4 \ &+ \ \frac{1}{2}(h_3 m - q_1 b_{13} - q_2 b_{23} - q_3 b_{33}) \ = \ 0 \, .
\end{aligned} \tag{6.6}
$$

We see that to cancel tadpoles the sign of the flux contribution must be opposite to that of D6-branes. We will now consider a AdS background with metric fluxes and $m \neq 0$ discussed in section 4.4. The reader can check that choosing the fluxes as

$$
q_i = q = h_i - 2 \, , \qquad a_i = 16 \, , \qquad m = b_{ij} = -b_{ii} = 4 \, , \tag{6.7}
$$

all RR tadpoles are cancelled. Note that eq. (4.32) fixes $h_0 = -12 h_i$, otherwise the values of $q$, $h_0$ and $h_i$ may be arbitrarily large still cancelling all RR tadpoles.

The above type of flux backgrounds does give rise to supersymmetric AdS vacua with all real moduli fixed. In fact, the fluxes in (6.7) are isotropic so that the superpotential is of the form (3.15) and the results of section 4.4 with $m \neq 0$ can be applied. One can easily check that with the above fluxes $\lambda_0 = 1 + (24/h_0)$, which is arbitrarily close to 1 for large $h_0$. Substituting these fluxes yields for the real moduli

$$
s = -\frac{h_0 \lambda}{96} t \, , \qquad u_k = -\frac{h_0 \lambda}{8} t \, , \qquad t = \sqrt{\frac{5}{3}} \frac{|h_0|}{16} \lambda^{1/2} \left( \lambda + \frac{24}{h_0} \right)^{1/2} \, , \tag{6.8}
$$

where $\lambda$ is the appropriate solution of eq. (4.40) for the $\lambda_0$ indicated above. For large $h_0$, $\lambda_0$ is close to 1 so that $\lambda \simeq (10)^{2/3}/20$ when $e_0 = c_1 = 0$. In this case one needs $h_0 < 0$. The imaginary part of the Kähler moduli are fixed as in eq. (4.39) whereas only the linear combination of dilaton and complex structure axions $12\mathrm{Im}S + \sum_{k=1}^{3} \mathrm{Im}U_k$ is fixed, as in eq. (4.33). As discussed in section 4.4, for large $h_0$ (which also implies large $h_k$, $q$) all moduli are stabilized in a regime in which perturbation theory in $D$=4 is a good approximation.

Note that the FW conditions (5.8) for the D6-branes $a, h_1$ and $h_2$ read respectively

$$
h_2 \ = \ h_3 \, , \qquad h_1 \ = \ h_3 \, , \qquad h_1 \ = \ h_2 \, , \tag{6.9}
$$

which are automatically satisfied because $h_1 = h_2 = h_3 = -h_0/12$. As we mentioned, this will guarantee that the the supersymmetry preserving conditions at the brane intersections

$$\begin{aligned} \mathrm{tg}^{-1}\left(\frac{\tau_2}{3}\right) - \mathrm{tg}^{-1}\left(\frac{\tau_3}{3}\right) &= 0\,, \\ \mathrm{tg}^{-1}\left(\frac{\tau_1}{2}\right) - \mathrm{tg}^{-1}\left(\frac{\tau_2}{2}\right) &= 0\,, \\ \mathrm{tg}^{-1}\left(\frac{\tau_1}{2}\right) - \mathrm{tg}^{-1}\left(\frac{\tau_3}{2}\right) &= 0\,, \end{aligned} \tag{6.10}$$

where $\tau_i = R^i_y/R^i_x$, are satisfied, since $u_1 = u_2 = u_3$. This is no surprise, since as we mentioned in section 5, in this class of AdS vacua all branes should be calibrated which in turn implies that the FW anomaly automatically cancels.

In this particular model it is also interesting to look at the structure of U(1)'s and the Im$U_I$ RR fields. It is easy to check that the couplings (5.1) give masses to two linear combinations of U(1)'s by combining with certain linear combinations of Im$U_I$ fields. Only the generator $Q_a - 2(Q_1 - Q_2)$ remains massless at this level. On the other hand, the fields Im$S$ and $\sum_k \mathrm{Im}U_k$ do not mix with the U(1)'s at all, as expected, since FW anomalies cancel. Note that the combination $12\mathrm{Im}S + \sum_k \mathrm{Im}U_k$ is the one which gets a mass from fluxes (see eq. (4.33)). The orthogonal linear combination is massless and may be identified with an axion which may be of relevance for the strong CP problem.

Although we have studied here only the dilaton and the diagonal closed string moduli of the orientifold, we already mentioned that setting all off-diagonal moduli to zero solves the extremum conditions. Furthermore, since we are in a $\mathcal{N}$=1 supersymmetric AdS background, this guarantees that these off-diagonal moduli are also stable. Thus, the closed string background discussed is completely stable. We have then succeeded in building the first semi-realistic $\mathcal{N}$=1 supersymmetric model with all closed string moduli stabilized in a consistent perturbative regime. The vacuum is AdS with a c.c. which may be made small (although not arbitrarily small, see below) for large fluxes. Unlike previous flux constructions in the present case we have a simple toroidal orientifold, without any further orbifold twist. Furthermore, the $\mathcal{N}$=1 supersymmetry conditions on the brane angles are forced upon us by the Freed-Witten constraint plus the minimization. Recall in this respect that in the $\mathcal{N}$=1 supersymmetric brane configurations constructed up to now the angles were fine-tuned to verify the supersymmetry conditions, there was no dynamical explanation for that choice, since not all closed string moduli were fixed.

Let us make some complementary comments about this kind of MSSM-like AdS constructions:

(i) Other MSSM-like models in AdS may be constructed along similar lines making use of the backgrounds with metric fluxes and $m \neq 0$ discussed in subsection 4.4. An easy way to proceed is to start with a tadpole free $\mathcal{N}$=1 MSSM-like D6-brane configuration and embed it in a AdS background of the type $\lambda_0 = 1$ in which fluxes do not contribute to RR tadpoles at all. For example, one can start again from the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold example in table 1 with $N_f = 5$ in which all RR tadpoles cancel. In the prescribed AdS background all moduli are stabilized in a perturbative regime for large enough fluxes.

(ii) One could think of building analogous AdS MSSM-like models with a background free of metric fluxes, as in subsection 4.3. This turns out to be difficult because fluxes contribute negatively to the RR tadpoles in eq. (3.17) and hence additional orientifold planes have to be added wrapping those directions. One possibility is to use again the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold which has such O6-planes, but the fluxes tend to overwhelm the contribution of orientifolds and this procedure does not look promising.

(iii) Besides the chiral spectrum described above, this class of toroidal models has massless adjoint chiral fields corresponding to the open string moduli parametrizing the location and Wilson lines on the branes. In a supersymmetric AdS background as the one we are considering here, those open string moduli are in any case stable. It is an interesting question to study what happens to them when some supersymmetry-breaking effect is included. It has been shown that fluxes in type IIB stabilize some (but not all) of the open string moduli in the toroidal case [56]. Additional ways to give masses to these degrees of freedom in toroidal models have been recently described in [57, 58]. It should be worth to study this question in the context of our type IIA AdS backgrounds.

(iv) Once all moduli are fixed in a given model like this, one can compute a number of interesting physical quantities like gauge coupling constants and Yukawa couplings, since they will be known functions of the fluxes. For example, in the above model the gauge kinetic functions of the groups SU(4), $\mathrm{SU}(2)_L$, $\mathrm{SU}(2)_R$ have

$$\mathrm{Re} f_{\mathrm{SU}(4)} \;=\; 9s + u_1\,; \qquad \mathrm{Re} f_{\mathrm{SU}(2)_L} \;=\; \frac{u_2}{2}\,; \qquad \mathrm{Re} f_{\mathrm{SU}(2)_R} \;=\; \frac{u_3}{2}\,. \tag{6.11}$$

Since these are the values at the string scale, to make contact with experiment we should then consider the running to low energies. As we said, simplest toroidal models like this have, in addition to the chiral spectrum, adjoint chiral fields which will generically spoil the running of coupling constants. Let us nevertheless proceed and compute them in this example. Since $u_k$ and $s$ are related by (4.31) one has e.g.

$$\alpha_{\mathrm{SU}(2)_L} \;=\; \frac{8}{\lambda |h_0| t} \;=\; \frac{128\sqrt{3}}{\sqrt{5} h_0^2 \lambda^{3/2} (\lambda + 24/h_0)^{1/2}}\,, \tag{6.12}$$

and $\alpha_{\mathrm{SU}(4)} = \frac{2}{7} \alpha_{\mathrm{SU}(2)_L}$. Thus, we see that the SM gauge coupling constants depend strongly on the fluxes. In particular we cannot make $h_0$ arbitrarily large (as one would naively do to decrease the value of the c.c.) since we would get then too small SM gauge couplings, inconsistent with experiment. For example, in the present model one can see that in order to get values $\alpha_i \simeq 1/5 - 1/30$ which might be consistent with low energy physics one needs to have $h_0 \simeq 100$ but not much bigger. This seems to be a generic property and not a particular feature of this class of models. Thus, indeed we have an infinite 'landscape' of models depending on unconstrained fluxes, but only a narrow region of fluxes would lead to consistent low-energy physics. Something similar happens with the Yukawa couplings, which have been computed and can be neatly written in terms of products of Jacobi $\vartheta$-functions in this model [54, 59]. They scale like the gauge couplings and hence are equally suppressed for large fluxes.

(v) These models are constructed in AdS and an obvious question is how one could promote this kind of vacua to dS. One possibility which comes to mind is to add anti-D6-branes. Indeed, if we add a pair of D6$-\overline{\rm D6}$ branes to the model, there is an extra contribution to the scalar potential which has the form

$$V_{\overline{D6}} \sim \frac{1}{u_1 u_2 u_3} \tag{6.13}$$

and should be included in the complete minimization. This would be essentially the mirror of the approach in [14] which was used in the type IIB case. More generally, one may consider sets of D6-branes with uncancelled NS-tadpoles. A potential is generated due to the missancellation of the tensions of the D6-branes against the orientifold tension, i.e. (in the string frame)

$$V_{\rm D6/O6} \;=\; \frac{T_6}{g_s}\left(\sum_a N_a \|l_a\| - \|l_{ori}\|\right) \;>\; 0\,, \tag{6.14}$$

where $\|l_a\|$ ($\|l_{ori}\|$) are the volume of the 3-cycles wrapped by each D6-brane (orientifold). It remains to be seen whether such a procedure can be made to work. In any event, the kind of fluxes considered here will stabilize non-supersymmetric D6-brane configurations with non-vanishing NS-tadpoles that have been considered in recent years. Examples of such non-supersymmetric D6-brane configurations with the chiral content of the SM are presented in appendix B.

## 7. Conclusions

In this paper we have studied the minima of the flux-induced effective moduli potential in a simple $\mathrm{T}^6/(\Omega(-1)^{F_L}I_3)$ IIA orientifold. We have focused on the dilaton and the diagonal Kähler and complex structure fiels, but we have nevertheless argued that the results found ignoring off-diagonal moduli still provide stable extrema of the full potential in relevant cases. We have considered RR, NS as well as metric background fluxes. Unlike the IIB case, the richness of the flux options leads to a full stabilization of all closed string moduli in AdS without the need of non-perturbative effects. Furthermore, the RR tadpole conditions, which are very restrictive in the IIB case, only constraint some flux combinations in the IIA case. Thus, there is enough freedom to adjust fluxes so that the minima are located in regions with large volume and small dilaton where the effective 4-dimensional supergravity approximations hold. The combination of metric fluxes with NS/RR fluxes leads to new possibilities such as fluxes fixing all moduli in $\mathcal{N}$=1 supersymmetric AdS but not contributing to RR tadpoles. This provides us with a rigid 'corset-like' background which can stabilize any RR tadpole-free D6-brane configuration in this toroidal setting. In general, if metric fluxes are turned on, the overall fluxes can contribute to RR tadpoles like O6-planes do, thereby providing the interesting possibility of disposing of orientifold planes in some cases.

In models with all real moduli fixed, only one linear combination of the axions of the dilaton and complex structure fields is determined at the minima. This, which at first sight appears to be a limitation of the approach, is in fact a blessing. Indeed, eventually we may like to add systems of D6-branes leading to chiral physics in the background. The RR axions which are never fixed by fluxes are in fact needed by the D6-branes to get rid of (potentially anomalous) open string U(1)'s. We have seen that cancellation of the Freed-Witten anomaly guarantees that sufficient axions remain to give Stückelberg masses to the U(1)'s. In the case of AdS $\mathcal{N}=1$ supersymmetric AdS vacua the cancellation of FW anomaly does in turn force the different sets of D6-branes to be calibrated.

One can construct explicit models with a chiral spectrum quite close to that of the MSSM with three generations and with all closed string moduli fixed in AdS. In its construction we make use of fluxes (including metric ones) contributing like orientifold planes to RR tadpoles. Other analogous models may also be built. The minima may be located at large volume and small dilaton so that we can trust our approximations. In such a model, with all moduli fixed, one can compute explicitly all gauge and Yukawa couplings as known functions of the fluxes undetermined by RR tadpoles. In the particular example of section 6.2, essentially only one flux (which may be identified with the NS 3-form flux $h_0$) fixes all couplings and scales. Thus, although one may talk about a landscape of models depending on a single flux parameter $h_0$, only a narrow region of integer values for $h_0$ would give rise to gauge couplings compatible with experimental constraints. In particular, in our concrete model there is not enough freedom to make the c.c. arbitrarily small by making $h_0$ large. We believe that this is quite a generic feature. The dilaton and complex structure fields which determine the (inverse of) gauge coupling constants grow like some power of the fluxes. If we make the fluxes too large in order to get e.g. a small c.c. the SM couplings would get far too small.

Our approach has been to consider the metric fluxes as a deformation added to the original torus. The resulting twisted torus is a non-Calabi-Yau manifold in which we still know the moduli and are able to introduce D6-branes. It would be interesting to go beyond the toroidal geometry in this spirit. In appendix A we have shown that the analysis of $\mathcal{N}=1$ vacua deduced from the effective flux-induced superpotential agrees with recent results on supersymmetric IIA compactifications on manifolds with SU(3) structure [60–66]. Although we have worked out many specific minima of the general fluxed potential in the simplest IIA toroidal orientifold, we cannot claim that we have done a complete analysis. Furthermore, we have not explored the possibilities offered by some of the solutions (e.g. AdS non-supersymmetric minima) that we have analyzed nor made a systematic search for MSSM-like D6-brane configurations. Presumably there are many other options beyond the ones that were discussed. All the models with all moduli stabilized are however AdS. An important problem is how to modify the premises in order to obtain models with dS vacua. A possible option is to add anti-D6-branes or, more generally, consider non-supersymmetric brane configurations. Positive definite contributions to the potential will then in general appear which might help in going to dS. We hope to come back to all these issues in the near future.

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## A. SU(3) structure of twisted torus

In this appendix we study the relation between the metric fluxes and the SU(3) structure of the twisted torus. The idea is to generalize the analysis of [22, 23] by turning on all metric fluxes in (2.22) and not only the $a_i$ obtained from T-duality of NS fluxes. We will see that in this more general situation the twisted torus is still a half-flat manifold as it occurs when only the $a_i$ are present [22, 23]. Using the results in this appendix we will also be able to describe our Minkowski and AdS supersymmetric vacua in terms of torsion classes. Supersymmetric IIA compactifications on manifolds with SU(3) structure have been recently considered in [60 – 66].

On the twisted torus one can build the fundamental 2-form $J$ and the holomorphic 3-form $\Omega$ in the usual way. Including the sizes and complex structure parameters we have

$$\begin{aligned} J &= -t_1\eta^1\wedge\eta^4 - t_2\eta^2\wedge\eta^5 - t_3\eta^3\wedge\eta^6\,, \\ \Omega &= (\eta^1 + i\tau_1\eta^4)\wedge(\eta^2 + i\tau_2\eta^5)\wedge(\eta^3 + i\tau_3\eta^6)\,. \end{aligned} \tag{A.1}$$

These forms define an SU(3) structure. In particular, they satisfy

$$J\wedge\Omega = 0\,, \qquad J\wedge J\wedge J = -\frac{3i}{4}\frac{t_1t_2t_3}{\tau_1\tau_2\tau_3}\Omega\wedge\Omega^*\,. \tag{A.2}$$

The torsion classes can be read from (see e.g. [22, 67, 60])

$$\begin{aligned} dJ &= \frac{3}{2}\frac{t_1t_2t_3}{\tau_1\tau_2\tau_3}\mathrm{Im}(\mathcal{W}_1\Omega^*) + \mathcal{W}_4\wedge J + \mathcal{W}_3\,, \\ d\Omega &= \mathcal{W}_1 J\wedge J + \mathcal{W}_2\wedge J + \mathcal{W}_5^*\wedge\Omega\,, \end{aligned} \tag{A.3}$$

where $\mathcal{W}_1$ is a complex 0-form, $\mathcal{W}_2$ is a primitive ($\mathcal{W}_2\wedge J\wedge J = 0$) complex 2-form, $\mathcal{W}_3$ is a primitive ($\mathcal{W}_3\wedge J = 0$) real $(2,1)\oplus(1,2)$-form, $\mathcal{W}_4$ is a real 1-form, and $\mathcal{W}_5$ is a complex $(1,0)$-form. The unusual factor in the first term of $dJ$ is needed so that $d(J\wedge\Omega) = 0$.

When only the metric fluxes (2.22) are turned on, using (2.16) we find

$$\begin{aligned} d\Omega &= \frac{1}{s}(a_1s + b_{11}u_1 + b_{12}u_2 + b_{13}u_3)\eta^{2536} + \frac{1}{s}(a_2s + b_{21}u_1 + b_{22}u_2 + b_{23}u_3)\eta^{1436} + \\ &\quad + \frac{1}{s}(a_3s + b_{31}u_1 + b_{32}u_2 + b_{33}u_3)\eta^{1425}\,, \\ dJ &= (a_1t_1 + a_2t_2 + a_3t_3)\eta^{456} - (b_{13}t_1 + b_{23}t_2 + b_{33}t_3)\eta^{126} - \\ &\quad - (b_{12}t_1 + b_{22}t_2 + b_{32}t_3)\eta^{153} - (b_{11}t_1 + b_{21}t_2 + b_{31}t_3)\eta^{423}\,, \end{aligned} \tag{A.4}$$

where $\eta^{153} = \eta^1\wedge\eta^5\wedge\eta^3$, etc. In $d\Omega$ we have used $u_i = s\tau_j\tau_k$, $i\neq j\neq k$.

Clearly, $d(J \wedge J) = 0$ and $d(\mathrm{Im}\Omega) = 0$, thus the twisted torus with the given fluxes is a half-flat manifold. For the torsion classes we easily read $\mathcal{W}_4 = \mathcal{W}_5 = 0$. The torsions $\mathcal{W}_2$ and $\mathcal{W}_3$ are different from zero, they can be readily obtained using (A.3) and

$$\mathcal{W}_1 = \frac{1}{6st_1t_2t_3}(a_1st_1 + a_2st_2 + a_3st_3 + b_{11}t_1u_1 + b_{12}t_1u_2 + b_{13}t_1u_3 + \\ + b_{21}t_2u_1 + b_{22}t_2u_2 + b_{23}t_2u_3 + b_{31}t_3u_1 + b_{32}t_3u_2 + b_{33}t_3u_3)\,. \quad (A.5)$$

Notice that this $\mathcal{W}_1$ is similar to $W_Q$ without the NS fluxes, as expected because it is basically computed as $\int \Omega \wedge dJ$.

In [62] it has been shown that supersymmetric Minkowski vacua of type IIA require $\mathcal{W}_1 = 0$. In our examples of this kind of vacua in section 4.1 we indeed find $\mathcal{W}_1 = 0$. This follows simply because taking real part of $\partial W/\partial \widetilde{U}_J = 0$ gives $\sum_i A_{iJ}t_i = 0$ which is enough to show $\mathcal{W}_1 = 0$. In fact, $dJ = 0$ so that $\mathcal{W}_3 = 0$ as well. We also find that $m = 0$ and then from the real part of $\partial W/\partial T_i = 0$ we deduce that $d\Omega = -\frac{1}{s}\overline{F}_2 \wedge J$. We have examples, such as $W(S, T_1, T_2, T_3)$, $W(U_1, T_1, T_2, T_3)$ or the $W(T_1, T_2, T_3, U_2, U_3)$ in (4.7), in which $d\Omega \neq 0$ and $\mathcal{W}_2 = -\frac{1}{s}\overline{F}_2$. Another characteristic feature of these models with $\overline{F}_2 \neq 0$ is the existence of flux tadpoles for some component of $C_7$. There are other models, such as $W(S, U_1, T_2, T_3)$ or $W(U_1, U_2, T_1, T_2)$, in which $d\Omega = 0$ and $C_7$ tadpoles vanish because $\overline{F}_2 = 0$ is required to have non-zero real parts of the moduli. In all cases, the typical configuration has neither NS fluxes nor RR fluxes for $F_6$ ($e_0 = 0$) and $F_4$ ($e_i = 0$). These results are in agreement with the analysis of [62, 63].

Type IIA supersymmetric AdS compactifications have also been studied in terms of SU(3) structures [64, 65]. It is interesting to see how the same type of results follows in our setup. To begin we notice that taking real part of $D_{\widetilde{U}_J}W = 0$ gives

$$\sum_{i=1}^{3} A_{iJ}t_i\widetilde{u}_J = -\frac{1}{2}\mathrm{Re}W\,, \qquad J = 0, \cdots, 3\,. \quad (A.6)$$

We can use these relations to compute $dJ$ and also

$$\mathcal{W}_1 = -\frac{\mathrm{Re}W}{3st_1t_2t_3}\,. \quad (A.7)$$

From the explicit $dJ$ we further read $\mathcal{W}_3 = 0$. To calculate $d\Omega$ we look instead at the real part of $D_{T_i}W = 0$ and deduce

$$a_is + b_{i1}u_1 + b_{i2}u_2 + b_{i3}u_3 = -\frac{\mathrm{Re}W}{2t_i} - \sum_{j\neq k\neq i} t_j(q_k + mv_k)\,. \quad (A.8)$$

Moreover, combining with (A.6) yields the relation

$$t_1t_2(q_3 + mv_3) + t_1t_3(q_2 + mv_2) + t_2t_3(q_1 + mv_1) = \frac{1}{4}\mathrm{Re}W\,. \quad (A.9)$$

It is then straightforward to determine $d\Omega$ and from it obtain

$$\mathcal{W}_2 \wedge J = -\frac{1}{4}\mathcal{W}_1 J \wedge J + \frac{m}{s}B_2 \wedge J - \frac{1}{s}\overline{F}_2 \wedge J\,, \quad (A.10)$$

| $N_i$ | $(n_i^1, m_i^1)$ | $(n_i^2, m_i^2)$ | $(n_i^3, m_i^3)$ |
|---|---|---|---|
| $N_a = 3$ | $(1/\beta^1, 0)$ | $(n_a^2, \epsilon\beta^2)$ | $(1/\rho, -\tilde{\epsilon}/2)$ |
| $N_b = 2$ | $(n_b^1, \tilde{\epsilon}\epsilon\beta^1)$ | $(1/\beta^2, 0)$ | $(1, -3\rho\tilde{\epsilon}/2)$ |
| $N_c = 1$ | $(n_c^1, 3\rho\epsilon\beta^1)$ | $(1/\beta^2, 0)$ | $(0, 1)$ |
| $N_d = 1$ | $(1/\beta^1, 0)$ | $(n_d^2, \epsilon\beta^2/\rho)$ | $(1, 3\rho\tilde{\epsilon}/2)$ |

**Table 4:** D6-brane wrapping numbers giving rise to a SM spectrum. The general solutions are parametrized by a phase $\epsilon, \tilde{\epsilon} = \pm 1$, the NS background on the first two tori $\beta^i = 1 - b^i = 1, 1/2$, four integers $n_a^2, n_b^1, n_c^1, n_d^2$ and a parameter $\rho = 1, 1/3$.

which satisfies $\mathcal{W}_2 \wedge J \wedge J = 0$ by virtue of (A.9). We conclude that supersymmetric AdS compactifications have $\mathcal{W}_1$ and $\mathcal{W}_2$ different from zero but $\mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$, as found in [65] in a more general setup. There is also a particular case in which $\mathcal{W}_2 = 0$ [64, 65]. Our example in section 4.4 is of this type. Indeed, using (4.31) gives $\mathcal{W}_1 = 2a/t^2$ and $d\Omega = \mathcal{W}_1 J \wedge J$. We also find $\overline{H}_3 = -\sqrt{|h_1 h_2 h_3/h_0|}\mathrm{Im}\Omega$.

## B. Stabilizing non-susy intersecting D-brane models

In the $\mathcal{N}$=1 supersymmetric AdS constructions in the main text, we have discussed examples in which all D6-branes preserve the same $\mathcal{N}$=1 supersymmetry. In this appendix we would like to study the non-supersymmetric class of semi-realistic intersecting D6-brane models of ref. [68]. One of the known problems of these non-susy models is that they are unstable due to the existence of NS tadpoles. These appear from a miscancellation of the tensions of the D6-branes with the orientifold tension. We would like to point out here that those non-susy models may in general become stable in the presence of fluxes. We will see that the FW conditions in AdS will force the branes to preserve supersymmetry locally, i.e. any pair of intersecting D6-branes will preserve one unbroken supersymmetry although there is no overall $\mathcal{N}$=1 supersymmetry preserved simultaneously by all D6-branes. We will see that all closed string moduli will be also determined, although, as we argue at the end, a complete treatment would require taking into account D-term's in the scalar potential.

In [68] a general class of solutions was given for the wrapping numbers $(n_a^i, m_a^i)$ giving rise to a SM spectrum. These are shown in table 4. In this table we have several discrete parameters. First we consider $\beta^i = 1, 1/2$. From the point of view of branes at angles $\beta^i = 1$ stands for a rectangular lattice for the $i^{th}$ torus, whereas $\beta^i = 1/2$ describes a tilted lattice allowed by the $\Omega I_3$ symmetry. We also have two phases $\epsilon, \tilde{\epsilon} = \pm 1$ and the parameter $\rho$ which can only take the values $\rho = 1, 1/3$. Furthermore, each of these families of D6-brane configurations depend on four integers ($n_a^2, n_b^1, n_c^1$ and $n_d^2$). Any of these choices leads exactly to the same massless fermion spectrum of the SM with 3 generations.

Now, imposing the condition (5.8) for branes $a$, $b$, $c$, $d$, respectively, leads to the relations

$$h_2 \, \frac{\epsilon\beta^2}{\rho\beta^1} \; - \; h_3 \, \frac{\tilde{\epsilon} n_a^2}{2\beta^1} \; = \; 0 \, ,$$

$$
\begin{aligned}
h_1 \, \frac{\epsilon\tilde{\epsilon}\beta^1}{\beta^2} \; - \; h_3 \, \frac{\tilde{\epsilon}3\rho n_b^1}{2\beta^2} \; &= \; 0\,, \\
h_3 \, \frac{n_c^1}{\beta^2} \; &= \; 0\,, \\
h_2 \, \frac{\epsilon\beta^2}{\rho\beta^1} \; + \; h_3 \, \frac{\tilde{\epsilon}3\rho n_d^2}{2\beta^1} \; &= \; 0\,.
\end{aligned}
\tag{B.1}
$$

These constraints have two solutions, depending on the value of $n_c^1$:

**(i)** $n_c^1 \neq 0$**.** In this case necessarily $h_3 = 0$ and hence $h_1 = h_2 = 0$. Only the flux $h_0$ may be added and only the field $S$ may be fixed by fluxes.

**(ii)** $n_c^1 = 0$**.** In this case one can check that the constraints are solved as long as

$$
\begin{aligned}
h_3 \; (n_a^2 + 3\rho n_d^2) \; &= 0\,, \\
h_1 \; &= \; \frac{3\rho n_b^1}{2\epsilon\beta^1} \, h_3\,, \\
h_2 \; &= \; \frac{\rho\tilde{\epsilon}n_a^2}{2\epsilon\beta^2} \, h_3\,,
\end{aligned}
\tag{B.2}
$$

so that one can only have non-vanishing $h_i$ if $n_a^2 = -3\rho n_d^2$. One can check that, when this condition is verified, there are two massless U(1)'s in the spectrum, $\mathrm{U}(1)_R$ and $\mathrm{U}(1)_{B-L}$, rather than just hypercharge, and only two linear combinations of the RR fields

$$
\mathrm{Im}U_2 \; - \; \frac{\tilde{\epsilon}\rho n_a^2}{2\epsilon\beta^2}\mathrm{Im}U_3\,, \qquad \mathrm{Im}U_1 \; - \; \frac{3\rho n_b^1}{2\epsilon\beta^1}\mathrm{Im}U_3
\tag{B.3}
$$

become massive by combining with the $\mathrm{U}(1)_{3B+L}$ and $\mathrm{U}(1)_b$ gauge bosons respectively. The orthogonal linear combination

$$
\frac{3\rho n_b^1}{2\epsilon\beta^1}\mathrm{Im}U_1 \; + \; \frac{\tilde{\epsilon}\rho n_a^2}{2\epsilon\beta^2}\mathrm{Im}U_2 \; + \; \mathrm{Im}U_3 \; = \; \frac{1}{h_3}\left(\sum_{I=1,2,3} h_I \mathrm{Im}U_I\right)
\tag{B.4}
$$

is precisely a piece of the combination appearing in the superpotential, which is expected to acquire a mass from fluxes upon minimization. The other RR field Im$S$ may become massive depending on the presence or not of a non-vanishing $h_0$ background, which leads to no constraint in the model (since $m_a^1 m_a^2 m_a^3 = 0$ for all the D6-branes present).

If upon minimization one finds $\langle \mathrm{Re}U_I \rangle \simeq 1/h_I$ one obtains that the above conditions imply

$$
\frac{\mathrm{Re}U_3}{\mathrm{Re}U_1} \; = \; \frac{3\rho n_b^1}{2\epsilon\beta^1}\,, \qquad \frac{\mathrm{Re}U_3}{\mathrm{Re}U_2} \; = \; \frac{\rho\tilde{\epsilon}n_a^2}{2\epsilon\beta^2}\,.
\tag{B.5}
$$

One can check that these conditions guarantee that at each brane intersection there is one unbroken supersymmetry, although in this model no overall supersymmetry generator is preserved by all intersections. This kind of local (but not global) supersymmetry was termed 'Q-SUSY' in [38]. Thus we see that adding fluxes $h_i \neq 0$, $i = 1, 2, 3$, is only possible if the brane configuration is locally supersymmetric.

Let us now be more explicit and chose the wrapping number parameters as follows:

$$n_a^2 = -n_d^2 = \beta_1 = \epsilon = \tilde{\epsilon} = 1\,; \qquad \beta_2 = 1/2\,, \quad \rho = 1/3\,, \qquad n_b^1 = 2\,. \tag{B.6}$$

One can easily check that RR tadpoles cancel without the addition of any further D6-brane nor fluxes. The conditions (B.2) now read

$$h_1 = h_3 = 3h_2\,. \tag{B.7}$$

Consider now again the same AdS vacua with $m \neq 0$ that we discussed in section 4.4. We saw there that for $m \neq 0$ one can find AdS vacua in which the NS/RR contribution $mh_0$ to RR tadpoles may be cancelled by the metric fluxes contribution $3ac_2$ ($\lambda_0 = 1$ case). Then we have the interesting possibility of fixing all closed string moduli without fluxes contributing to RR tadpoles at all. Consider backgrounds as follows

$$b_k = (-6, -2, -6)\,; \qquad h_k = rb_k\,, \tag{B.8}$$

where $r = -h_0/3a$ must be a positive integer. Then, if we further chose $e_0 = c_1 = 0$ one finds a minimum of the flux-induced potential as long as $h_0 > 0$, $a < 0$, $m < 0$. The flux contribution to tadpoles vanishes if we further take $c_2 = -rm$. Then the real parts of closed string moduli are fixed as

$$u_k = \frac{3as}{b_k}\,, \qquad s = \frac{m}{a10^{1/3}}rt\,; \qquad t \;=\; \frac{\sqrt{15}10^{2/3}}{20}r\,. \tag{B.9}$$

Certain linear combinations of the imaginary parts of the moduli fields are fixed as discussed in subsection 4.4. Note that choosing $r = -h_0/3a$ large one can fix the moduli at arbitrarily large values with small 4- and 10-dimensional dilatons.

As we mentioned, one has to be careful in applying the results obtained in the main text to a non-supersymmetric brane configuration like this. Indeed, in this case in addition to the F-term scalar potential one has to add the piece (6.14). Still one expects a full determination of all closed string moduli also in this non-supersymmetric example.

## References


[1] S. Gukov, C. Vafa and E. Witten, *CFT's from Calabi-Yau four-folds*, *Nucl. Phys.* **B 584** (2000) 69 [hep-th/9906070].

[2] S.B. Giddings, S. Kachru and J. Polchinski, *Hierarchies from fluxes in string compactifications*, *Phys. Rev.* **D 66** (2002) 106006 [hep-th/0105097].

[3] S. Kachru, M.B. Schulz and S. Trivedi, *Moduli stabilization from fluxes in a simple IIB orientifold*, *JHEP* **10** (2003) 007 [hep-th/0201028].

[4] A.R. Frey and J. Polchinski, $N = 3$ *warped compactifications*, *Phys. Rev.* **D 65** (2002) 126009 [hep-th/0201029].

[5] P.K. Tripathy and S.P. Trivedi, *Compactification with flux on K3 and tori*, *JHEP* **03** (2003) 028 [hep-th/0301139].

[6] R. Blumenhagen, D. Lüst and T.R. Taylor, *Moduli stabilization in chiral type-IIB orientifold models with fluxes*, *Nucl. Phys.* **B 663** (2003) 319 [hep-th/0303016].

[7] J.F.G. Cascales and A.M. Uranga, *Chiral 4d $N = 1$ string vacua with D-branes and NS-NS and RR fluxes*, *JHEP* **05** (2003) 011 [hep-th/0303024]; *Chiral 4d string vacua with D-branes and moduli stabilization*, hep-th/0311250.

[8] M. Berg, M. Haack and B. Körs, *An orientifold with fluxes and branes via t-duality*, *Nucl. Phys.* **B 669** (2003) 3 [hep-th/0305183]; *Brane/flux interactions in orientifolds*, *Fortschr. Phys.* **52** (2004) 583 [hep-th/0312172].

[9] S. Ashok and M.R. Douglas, *Counting flux vacua*, *JHEP* **01** (2004) 060 [hep-th/0307049];
A. Giryavets, S. Kachru, P.K. Tripathy and S.P. Trivedi, *Flux compactifications on Calabi-Yau threefolds*, *JHEP* **04** (2004) 003 [hep-th/0312104];
F. Denef and M.R. Douglas, *Distributions of flux vacua*, *JHEP* **05** (2004) 072 [hep-th/0404116];
A. Giryavets, S. Kachru and P.K. Tripathy, *On the taxonomy of flux vacua*, *JHEP* **08** (2004) 002 [hep-th/0404243];
A. Misra and A. Nanda, *Flux vacua statistics for two-parameter Calabi-Yau's*, *Fortschr. Phys.* **53** (2005) 246 [hep-th/0407252];
J.P. Conlon and F. Quevedo, *On the explicit construction and statistics of Calabi-Yau flux vacua*, *JHEP* **10** (2004) 039 [hep-th/0409215].

[10] D. Lüst, S. Reffert and S. Stieberger, *Flux-induced soft supersymmetry breaking in chiral type-IIB orientifolds with d3/d7-branes*, *Nucl. Phys.* **B 706** (2005) 3 [hep-th/0406092].

[11] F. Marchesano and G. Shiu, *MSSM vacua from flux compactifications*, *Phys. Rev.* **D 71** (2005) 011701 [hep-th/0408059]; *Building MSSM flux vacua*, *JHEP* **11** (2004) 041 [hep-th/0409132].

[12] M. Cvetič and T. Liu, *Supersymmetric standard models, flux compactification and moduli stabilization*, *Phys. Lett.* **B 610** (2005) 122 [hep-th/0409032];
M. Cvetič, T. Li and T. Liu, *Standard-like models as type-IIB flux vacua*, *Phys. Rev.* **D 71** (2005) 106008 [hep-th/0501041].

[13] A. Font, *$\mathbb{Z}_n$ orientifolds with flux*, *JHEP* **11** (2004) 077 [hep-th/0410206].

[14] S. Kachru, R. Kallosh, A. Linde and S.P. Trivedi, *De Sitter vacua in string theory*, *Phys. Rev.* **D 68** (2003) 046005 [hep-th/0301240].

[15] F. Denef, M.R. Douglas and B. Florea, *Building a better racetrack*, *JHEP* **06** (2004) 034 [hep-th/0404257];
V. Balasubramanian, P. Berglund, J.P. Conlon and F. Quevedo, *Systematics of moduli stabilisation in Calabi-Yau flux compactifications*, *JHEP* **03** (2005) 007 [hep-th/0502058];
F. Denef, M.R. Douglas, B. Florea, A. Grassi and S. Kachru, *Fixing all moduli in a simple F-theory compactification*, hep-th/0503124;
P. Berglund and P. Mayr, *Non-perturbative superpotentials in F-theory and string duality*, hep-th/0504058;
J.P. Conlon, F. Quevedo and K. Suruliz, *Large-volume flux compactifications: moduli spectrum and D3/D7 soft supersymmetry breaking*, hep-th/0505076.

[16] I. Antoniadis and T. Maillard, *Moduli stabilization from magnetic fluxes in type-I string theory*, *Nucl. Phys.* **B 716** (2005) 3 [hep-th/0412008];
M. Bianchi and E. Trevigne, *The open story of the magnetic fluxes*, hep-th/0502147;

I. Antoniadis, A. Kumar and T. Maillard, *Moduli stabilization with open and closed string fluxes*, hep-th/0505260.

[17] J.P. Derendinger, C. Kounnas, P.M. Petropoulos and F. Zwirner, *Superpotentials in iia compactifications with general fluxes*, *Nucl. Phys.* **B 715** (2005) 211 [hep-th/0411276]; *Fluxes and gaugings: $N = 1$ effective superpotentials*, *Fortschr. Phys.* **53** (2005) 926 [hep-th/0503229].

[18] S. Kachru and A.K. Kashani-Poor, *Moduli potentials in type-IIA compactifications with RR and ns flux*, *JHEP* **03** (2005) 066 [hep-th/0411279].

[19] T.W. Grimm and J. Louis, *The effective action of type-IIA Calabi-Yau orientifolds*, *Nucl. Phys.* **B 718** (2005) 153 [hep-th/0412277].

[20] G. Villadoro and F. Zwirner, *N = 1 effective potential from dual type-IIA D6/O6 orientifolds with general fluxes*, *JHEP* **06** (2005) 047 [hep-th/0503169].

[21] O. DeWolfe, A. Giryavets, S. Kachru and W. Taylor, *Type IIA moduli stabilization*, *JHEP* **07** (2005) 066 [hep-th/0505160].

[22] S. Gurrieri, J. Louis, A. Micu and D. Waldram, *Mirror symmetry in generalized Calabi-Yau compactifications*, *Nucl. Phys.* **B 654** (2003) 61 [hep-th/0211102].

[23] S. Kachru, M.B. Schulz, P.K. Tripathy and S.P. Trivedi, *New supersymmetric string compactifications*, *JHEP* **03** (2003) 061 [hep-th/0211182].

[24] M.B. Schulz, *Superstring orientifolds with torsion: O5 orientifolds of torus fibrations and their massless spectra*, *Fortschr. Phys.* **52** (2004) 963 [hep-th/0406001].

[25] J. Scherk and J.H. Schwarz, *Spontaneous breaking of supersymmetry through dimensional reduction*, *Phys. Lett.* **B 82** (1979) 60; *How to get masses from extra dimensions*, *Nucl. Phys.* **B 153** (1979) 61.

[26] N. Kaloper and R.C. Myers, *The O(dd) story of massive supergravity*, *JHEP* **05** (1999) 010 [hep-th/9901045].

[27] G. Dall'Agata and S. Ferrara, *Gauged supergravity algebras from twisted tori compactifications with fluxes*, *Nucl. Phys.* **B 717** (2005) 223 [hep-th/0502066].

[28] L. Andrianopoli, M.A. Lledó and M. Trigiante, *The Scherk-Schwarz mechanism as a flux compactification with internal torsion*, *JHEP* **05** (2005) 051 [hep-th/0502083].

[29] C.M. Hull and R.A. Reid-Edwards, *Flux compactifications of string theory on twisted tori*, hep-th/0503114.

[30] B.S. Acharya, F. Denef, C. Hofman and N. Lambert, *Freund-Rubin revisited*, hep-th/0308046.

[31] J.F.G. Cascales and A.M. Uranga, *Branes on generalized calibrated submanifolds*, *JHEP* **11** (2004) 083 [hep-th/0407132].

[32] J.F.G. Cascales, *Flux compactifications: theory and model building*, Ph.D. Thesis, unpublished.

[33] D.S. Freed and E. Witten, *Anomalies in string theory with D-branes*, hep-th/9907189.

[34] J.M. Maldacena, G.W. Moore and N. Seiberg, *D-brane instantons and K-theory charges*, *JHEP* **11** (2001) 062 [hep-th/0108100].

[35] M. Bodner, A.C. Cadavid and S. Ferrara, *(2,2) vacuum configurations for type-IIA superstrings: $N = 2$ supergravity lagrangians and algebraic geometry*, *Class. and Quant. Grav.* **8** (1991) 789.

[36] J. Polchinski and A. Strominger, *New vacua for type-II string theory*, *Phys. Lett.* **B 388** (1996) 736 [hep-th/9510227].

[37] J. Louis and A. Micu, *Type II theories compactified on Calabi-Yau threefolds in the presence of background fluxes*, *Nucl. Phys.* **B 635** (2002) 395 [hep-th/0202168].

[38] D. Cremades, L.E. Ibáñez and F. Marchesano, *SUSY quivers, intersecting branes and the modest hierarchy problem*, *JHEP* **07** (2002) 009 [hep-th/0201205].

[39] J. Polchinski, *Dirichlet-branes and Ramond-Ramond charges*, *Phys. Rev. Lett.* **75** (1995) 4724 [hep-th/9510017].

[40] R. Bousso and J. Polchinski, *Quantization of four-form fluxes and dynamical neutralization of the cosmological constant*, *JHEP* **06** (2000) 006 [hep-th/0004134].

[41] T.R. Taylor and C. Vafa, *RR flux on Calabi-Yau and partial supersymmetry breaking*, *Phys. Lett.* **B 474** (2000) 130 [hep-th/9912152].

[42] I.V. Lavrinenko, H. Lu and C.N. Pope, *Fibre bundles and generalised dimensional reductions*, *Class. and Quant. Grav.* **15** (1998) 2239 [hep-th/9710243].

[43] S. Hellerman, J. McGreevy and B. Williams, *Geometric constructions of nongeometric string theories*, *JHEP* **01** (2004) 024 [hep-th/0208174];
A. Dabholkar and C. Hull, *Duality twists, orbifolds and fluxes*, *JHEP* **09** (2003) 054 [hep-th/0210209];
C.M. Hull and A. Catal-Ozer, *Compactifications with S-duality twists*, *JHEP* **10** (2003) 034 [hep-th/0308133];
A. Flournoy, B. Wecht and B. Williams, *Constructing nongeometric vacua in string theory*, *Nucl. Phys.* **B 706** (2005) 127 [hep-th/0404217];
C.M. Hull, *A geometry for non-geometric string backgrounds*, hep-th/0406102.

[44] A.M. Uranga, *D-brane, fluxes and chirality*, *JHEP* **04** (2002) 016 [hep-th/0201221].

[45] G.L. Cardoso, G. Curio, G. Dall'Agata and D. Lüst, *BPS action and superpotential for heterotic string compactifications with fluxes*, *JHEP* **10** (2003) 004 [hep-th/0306088];
*Heterotic string theory on non-kaehler manifolds with h- flux and gaugino condensate*, *Fortschr. Phys.* **52** (2004) 483 [hep-th/0310021];
K. Becker, M. Becker, K. Dasgupta and S. Prokushkin, *Properties of heterotic vacua from superpotentials*, *Nucl. Phys.* **B 666** (2003) 144 [hep-th/0304001];
S. Gurrieri, A. Lukas and A. Micu, *Heterotic on half-flat*, *Phys. Rev.* **D 70** (2004) 126009 [hep-th/0408121].

[46] G. Lopes Cardoso, D. Lüst and T. Mohaupt, *Modular symmetries of $N = 2$ black holes*, *Phys. Lett.* **B 388** (1996) 266 [hep-th/9608099].

[47] G. Curio, A. Klemm, D. Lüst and S. Theisen, *On the vacuum structure of type-II string compactifications on Calabi-Yau spaces with H-fluxes*, *Nucl. Phys.* **B 609** (2001) 3 [hep-th/0012213].

[48] E. Cremmer, S. Ferrara, C. Kounnas and D.V. Nanopoulos, *Naturally vanishing cosmological constant in $N = 1$ supergravity*, *Phys. Lett.* **B 133** (1983) 61.

[49] P. Breitenlohner and D.Z. Freedman, *Stability in gauged extended supergravity*, *Ann. Phys. (NY)* **144** (1982) 249.

[50] L.E. Ibáñez, R. Rabadán and A.M. Uranga, *Anomalous* U(1)*'s in type-I and type-IIB* $D = 4$, $N = 1$ *string vacua*, *Nucl. Phys.* **B 542** (1999) 112 [hep-th/9808139];
G. Aldazabal, S. Franco, L.E. Ibáñez, R. Rabadán and A.M. Uranga, $D = 4$ *chiral string compactifications from intersecting branes*, *J. Math. Phys.* **42** (2001) 3103 [hep-th/0011073].

[51] M. Cvetič, G. Shiu and A.M. Uranga, *Three-family supersymmetric standard like models from intersecting brane worlds*, *Phys. Rev. Lett.* **87** (2001) 201801 [hep-th/0107143];
*Chiral four-dimensional* $N = 1$ *supersymmetric type-IIA orientifolds from intersecting d6-branes*, *Nucl. Phys.* **B 615** (2001) 3 [hep-th/0107166].

[52] M. Cvetič, P. Langacker, T.J. Li and T. Liu, *D6-brane splitting on type-IIA orientifolds*, *Nucl. Phys.* **B 709** (2005) 241 [hep-th/0407178].

[53] D. Cremades, L.E. Ibáñez and F. Marchesano, *More about the standard model at intersecting branes*, hep-ph/0212048.

[54] D. Cremades, L.E. Ibáñez and F. Marchesano, *Yukawa couplings in intersecting D-brane models*, *JHEP* **07** (2003) 038 [hep-th/0302105]; *Computing Yukawa couplings from magnetized extra dimensions*, *JHEP* **05** (2004) 079 [hep-th/0404229].

[55] E. Dudas and C. Timirgaziu, *Internal magnetic fields and supersymmetry in orientifolds*, *Nucl. Phys.* **B 716** (2005) 65 [hep-th/0502085].

[56] P.G. Cámara, L.E. Ibáñez and A.M. Uranga, *Flux-induced SUSY-breaking soft terms*, *Nucl. Phys.* **B 689** (2004) 195 [hep-th/0311241]; *Flux-induced SUSY-breaking soft terms on d7-d3 brane systems*, *Nucl. Phys.* **B 708** (2005) 268 [hep-th/0408036];
M. Graña, T.W. Grimm, H. Jockers and J. Louis, *Soft supersymmetry breaking in Calabi-Yau orientifolds with D-branes and fluxes*, *Nucl. Phys.* **B 690** (2004) 21 [hep-th/0312232];
L. Görlich, S. Kachru, P.K. Tripathy and S.P. Trivedi, *Gaugino condensation and nonperturbative superpotentials in flux compactifications*, *JHEP* **12** (2004) 074 [hep-th/0407130];
D. Lüst, P. Mayr, S. Reffert and S. Stieberger, *F-theory flux, destabilization of orientifolds and soft terms on D7-branes*, hep-th/0501139.

[57] R. Blumenhagen, M. Cvetič, F. Marchesano and G. Shiu, *Chiral D-brane models with frozen open string moduli*, *JHEP* **03** (2005) 050 [hep-th/0502095].

[58] C. Angelantonj, M. Cardella and N. Irges, *Scherk-Schwarz breaking and intersecting branes*, hep-th/0503179.

[59] M. Cvetič and I. Papadimitriou, *Conformal field theory couplings for intersecting D-branes on orientifolds*, *Phys. Rev.* **D 68** (2003) 046001 [hep-th/0303083];
S.A. Abel and A.W. Owen, *Interactions in intersecting brane models*, *Nucl. Phys.* **B 663** (2003) 197 [hep-th/0303124]; *N-point amplitudes in intersecting brane models*, *Nucl. Phys.* **B 682** (2004) 183 [hep-th/0310257].

[60] J.P. Gauntlett, D. Martelli and D. Waldram, *Superstrings with intrinsic torsion*, *Phys. Rev.* **D 69** (2004) 086002 [hep-th/0302158].

[61] G. Dall'Agata and N. Prezas, $N = 1$ *geometries for M-theory and type-IIA strings with fluxes*, *Phys. Rev.* **D 69** (2004) 066004 [hep-th/0311146].

[62] M. Graña, R. Minasian, M. Petrini and A. Tomasiello, *Supersymmetric backgrounds from generalized Calabi-Yau manifolds*, *JHEP* **08** (2004) 046 [hep-th/0406137].

[63] P. Kaste, R. Minasian, M. Petrini and A. Tomasiello, *Kaluza-Klein bundles and manifolds of exceptional holonomy*, *JHEP* **09** (2002) 033 [hep-th/0206213];
*Nontrivial RR two-form field strength and* SU(3)*-structure*, *Fortschr. Phys.* **51** (2003) 764 [hep-th/0301063].

[64] K. Behrndt and M. Cvetič, *Supersymmetric intersecting D6-branes and fluxes in massive type-IIA string theory*, *Nucl. Phys.* **B 676** (2004) 149 [hep-th/0308045]; *General $N = 1$ supersymmetric flux vacua of (massive) type-IIA string theory*, hep-th/0403049; *General $N = 1$ supersymmetric fluxes in massive type-IIA string theory*, *Nucl. Phys.* **B 708** (2005) 45 [hep-th/0407263].

[65] D. Lüst and D. Tsimpis, *Supersymmetric $AdS_4$ compactifications of iia supergravity*, *JHEP* **02** (2005) 027 [hep-th/0412250].

[66] T. House and E. Palti, *Effective action of (massive) IIA on manifolds with* SU(3) *structure*, *Phys. Rev.* **D 72** (2005) 026004 [hep-th/0505177].

[67] G.L. Cardoso et al., *Non-Kähler string backgrounds and their five torsion classes*, *Nucl. Phys.* **B 652** (2003) 5 [hep-th/0211118].

[68] L.E. Ibáñez, F. Marchesano and R. Rabadán, *Getting just the standard model at intersecting branes*, *JHEP* **11** (2001) 002 [hep-th/0105155].